Project 2

(due by 12/21/05 – 10:30am)

Discuss the problem below in a concise and precise essay, at most 5 typed pages long. Whenever you use a reference, quote it and do not copy. Use your own words.

Consider the linear operator $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ which has the matrix

$$\begin{bmatrix} -4 & 3 & 1 & -1 \\ -6 & 5 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & -1 & 4 \end{bmatrix}$$

with respect to the standard basis.

1. Argue that there is a basis v_1, v_2, v_3, v_4 of $\mathbb{R}^4$ for which the matrix of T is upper-triangular by explicit construction, using the methods of Chapter 5. In particular, all eigenvalues are real. Give them and compute the matrix.

2. Find all invariant subspaces of T. Why is there no basis of eigenvectors, so T is not diagonalizable?

3. Finally, apply the Gram-Schmidt process to the above basis and construct an orthonormal basis e_1, e_2, e_3, e_4 and the corresponding upper-triangular matrix for T according to Schur's Theorem.