

MAT 539: Algebraic Topology, Homework Problems

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- (1) Draw the image of a system of parallel lines under the inversion $z \mapsto 1/z$. Prove it.
- (2) Let

$$P(z) = y^n + b_1(z)y^{n-1} + b_2(z)y^{n-2} \cdots + b_{n-1}(z)y + b_0(z)$$

be a monic polynomial of degree n on y , such that the coefficients $b_i(z)$ are polynomial functions of z . Prove that if $P(z)$ has n distinct roots for some value of z then there are also n distinct roots for all but finitely many values of z . (Hint: Use the discriminant. Example: If $P(z) = y^2 + b(z)y + c(z)$ then the discriminant equals $(b(z))^2 - 4c(z)$).

- (3) Make a picture of the three-sheeted surface cover of the completed z -plane associated to the equation $y^3 = (z - a)(z - b)(z - c)$, where a, b and c are distinct complex numbers (remember that the completion is done by adding into the surface the appropriate number of points over the points of infinity of the z -plane)
- (4) What is the multiple connectivity of a surface of genus two? Draw a sequence of pictures like the ones made in class going from the torus to a disk for the surface of genus two going to a disk.