Midterm 2

MAT 515, Fall 2019

November 13, 2019

Stony Brook University

Name: **ID #**:

(please print)

	1	2	3	4	5	**Total**
	12pts	8pts	8pts	12pts	10pts	50pts
Grade						

No notes or books.

You must provide explanation, not just the answer. Answers without justification will get only partial credit.

Please cross out anything that is not a part of your solution — e.g., some preliminary computations that you didn't need.

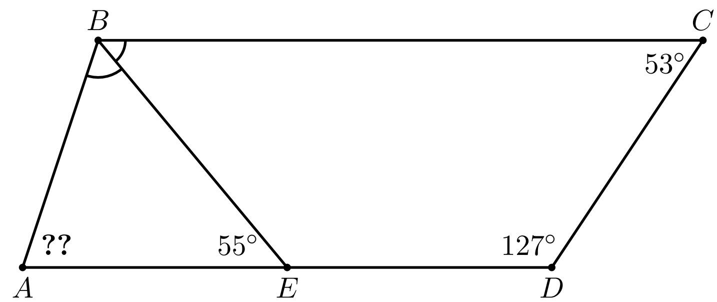
Instructor: Dzmitry Dudko

1. *(6+6 pts)*

(a) Let $ABCD$ be a convex quadrilateral, and let E be a point on AD such that BE bisects the angle $\angle ABC$. Suppose that

$$\angle DCB = 53^\circ, \quad \angle ADC = 127^\circ, \quad \angle AEB = 55^\circ.$$

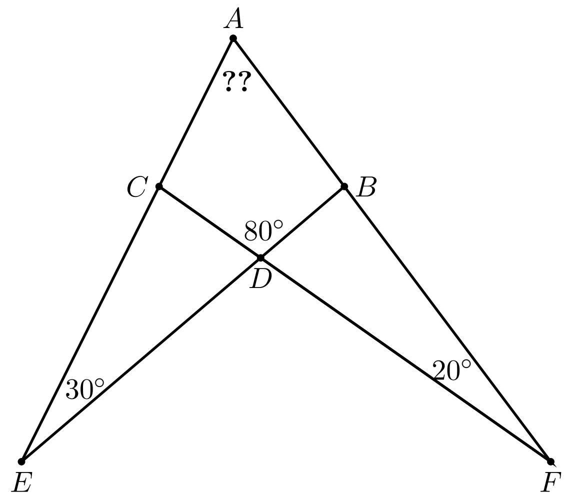
Find $\angle BAD$.
Hint: compute all angles.



Answer: $\angle BAD = 70^\circ$. □

(b) Suppose that a convex quadrilateral $ABCD$ is the intersection of $\triangle ABE$ and $\triangle ACF$ as it shown on the figure below. Compute $\angle BAC$ if

$$\angle AEB = 30^\circ, \quad \angle AFC = 20^\circ, \quad \angle CDB = 80^\circ.$$



Answer: $\angle BAC = 30^\circ$. □

2. *(3+5 pts)*

(a) Let $ABCD$ be a convex quadrilateral such that $\triangle ABC$ and $\triangle DAC$ are equilateral triangles. Show that $ABCD$ is a parallelogram.

(b) The vertices of a parallelogram $ABCD$ are on a circle. Show that $ABCD$ is a rectangle.

Solution.

(a) Since $AB = BC = AC = CD = DA$, the quadrilateral $ABCD$ is a rhombus. In particular, $ABCD$ is a parallelogram.

(b) Since $ABCD$ is a parallelogram, $\angle ABC = \angle CDA$. On the other hand $\angle ABC + \angle CDA = 180°$ because $\angle ABC + \angle CDA$ is $\frac{1}{2}\left(A\overset{\frown}{B}C + C\overset{\frown}{D}A\right) = \frac{1}{2}360° = 180°$ – compare with Problem 1 of HW 10.

Therefore, $\angle ABC = \angle CDA = 90°$.

□

3. *(3+5 pts)*

(a) Using a compass and a straightedge, construct an equilateral triangle ABC given the sum $AB + BC$.

(b) Using a compass and a straightedge, construct an equilateral triangle ABC given the length of the altitude belonging to the vertex A.

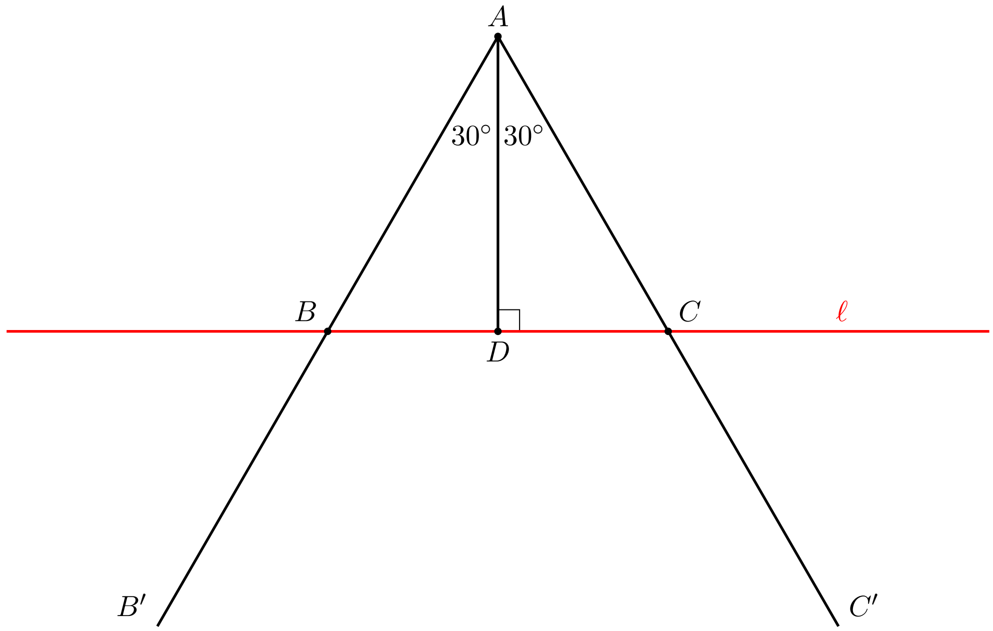
Solution.

(a) Bisect the given segment $AB + BC$ into two equal segments and construct a required equilateral triangle.

Recall: to construct an equilateral triangle $\triangle ABC$ given its side AB, we draw two circles centered at A and B of radius AB. Then C is one of the intersection points of the circles.

(b) Since we can construct an equilateral triangle, we can construct a $60°$-angle. Bisecting a $60°$-angle, we obtain a $30°$-angle.

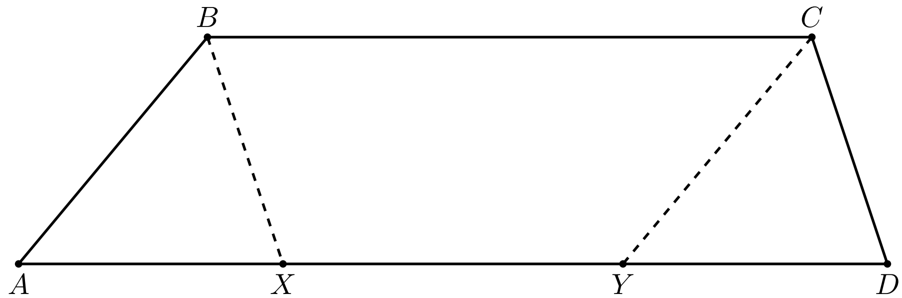
Construct an altitude AD of the given length. At D construct a line ℓ perpendicular to AD. Construct half-lines AC' and AB' that have $30°$-angle with AD. Let B and C be the intersections of AB' and AC' with ℓ. Then $\triangle ABC$ is a required triangle.



□

4. *(4+5+3 pts)*

(a) Let $ABCD$ be a trapezoid where $BC < AD$ are its bases (i.e., $BC||AD$ – parallel sides). Let X and Y be two points on AD such that $BX||CD$ and $CY||BA$. Prove that $\triangle ABX$ and $\triangle YCD$ are congruent.



Solution. Since $ABCY$ is a parallelogram, $AB = CY$ and $AY = BC$. Since $BXDC$ is a parallelogram, $BX = CD$ and $BC = XD$. Since $AY = BC = XD$, we have $AX = YD$. Therefore, $\triangle ABX = \triangle YCD$ by SSS.

□

(b) Let $ABCD$ and $A'B'C'D'$ be two trapezoid where $BC < AD$ and $B'C' < A'D'$ are their bases. Suppose that

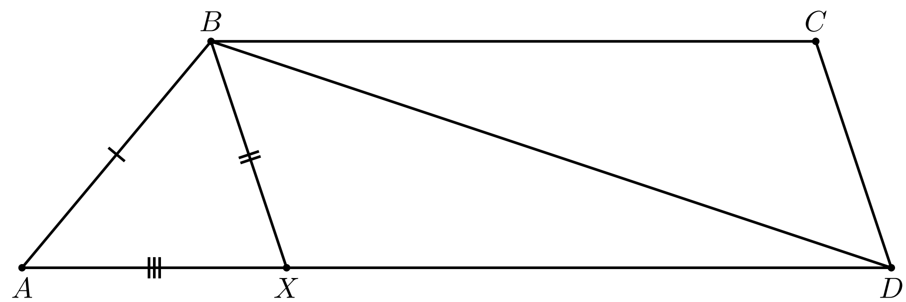
$$AB = A'B',\ BC = B'C',\ CD = C'D',\ DA = D'A'.$$

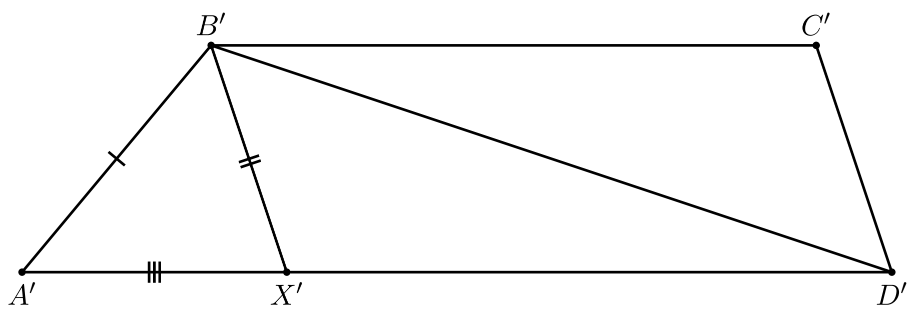
Prove that $ABCD$ and $A'B'C'D'$ are congruent.

(c) Let $ABCD$ be a trapezoid where $BC < AD$ are its bases. Prove that if $AB = DC$, then $\angle BAD = \angle CDA$.

Solution.

(b) As in **(a)**, construct BX parallel and congruent to CD, and construct $B'X'$ parallel and congruent to $C'D'$. Then $\triangle ABX = \triangle A'B'X'$ by SSS. This implies that $\angle BXD = \angle B'X'D'$. Then $\triangle BXD = \triangle B'X'D'$ by SAS. Finally, $\triangle BCD = \triangle B'C'D'$ by SSS.

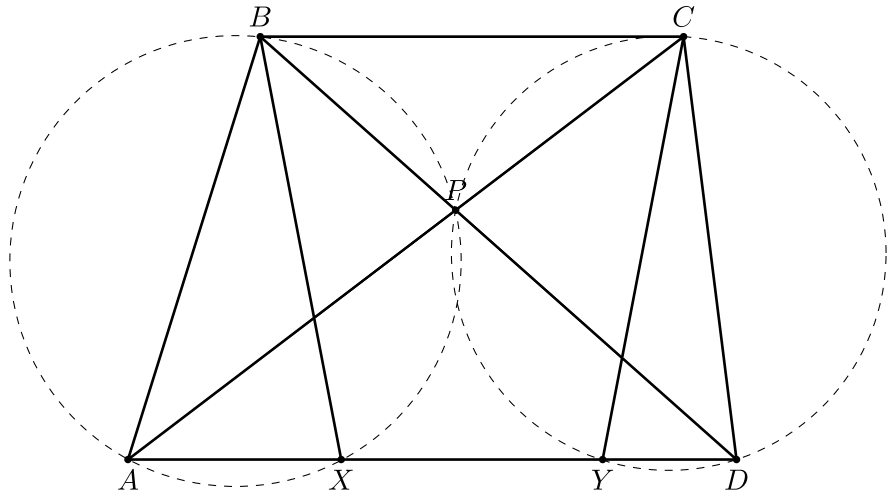




(c) Using **(b)**, the trapezoid $ABCD$ is congruent to $DCBA$. Therefore, $\angle BAD = \angle CDA$. □

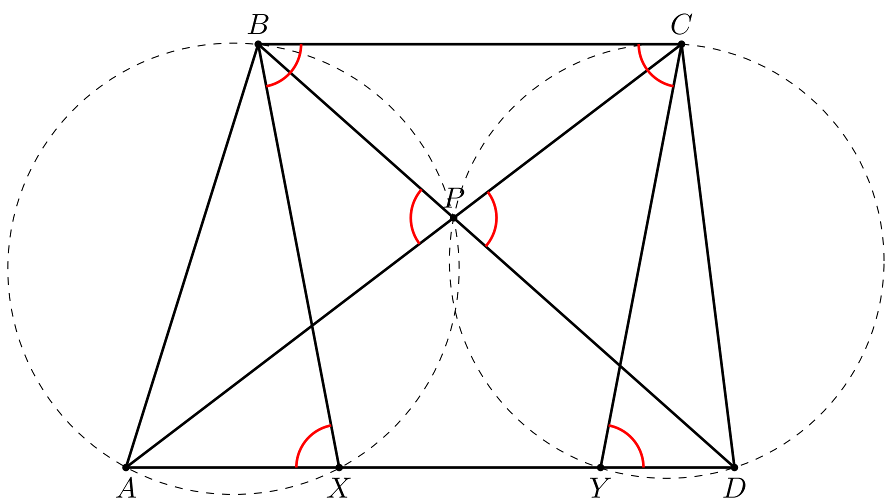
5. *(10 pts)*

Let $ABCD$ be a trapezoid where $BC < AD$ are its bases (i.e., $BC||AD$). Let P be the intersection of the diagonals AC and BD. Assume that X and Y are points on AD such that there is a circle containing A, B, P, X and there is a circle containing D, C, P, Y.
Prove that $\angle CBX = \angle APB = \angle BCY$.



Solution. We have:

- $\angle CBX = \angle AXB$ because $BC||AD$,
- $\angle AXB = \angle APB = \frac{1}{2}\overset{\frown}{BA}$,
- $\angle APB = \angle CPD$ – vertical angles,
- $\angle CPD = \angle DYC = \frac{1}{2}\overset{\frown}{CD}$,
- $\angle DYC = \angle BCY$ because $BC||AD$.



□