Midterm 1
MAT 515, Fall 2019
October 2, 2019
Stony Brook University

Name: (please print)	**ID #**:

	1 8pt	2 10pt	3 10pt	4 10pt	5 12pt	**Total** 50pts
Grade						

No notes or books.

You must provide explanation, not just the answer. Answers without justification will get only partial credit.

Please cross out anything that is not a part of your solution — e.g., some preliminary computations that you didn't need.

Instructor: Dzmitry Dudko

1. *(2+2+4 pts)*
Recall that $1^\circ = 60'$ and $1' = 60''$.

(a) Is the sum of the angles $34^\circ 34' 34''$ and $55^\circ 55' 55''$ acute or obtuse?
(b) Is the difference of the angles $134^\circ 34' 34''$ and $55^\circ 55' 55''$ acute or obtuse?
(c) Three lines passing through a given point divide the plane into six angles. Two of these angles turn out to be measuring $35^\circ 30'$ and $54^\circ 30'$. Find the measure of the remaining four angles.

Answers:

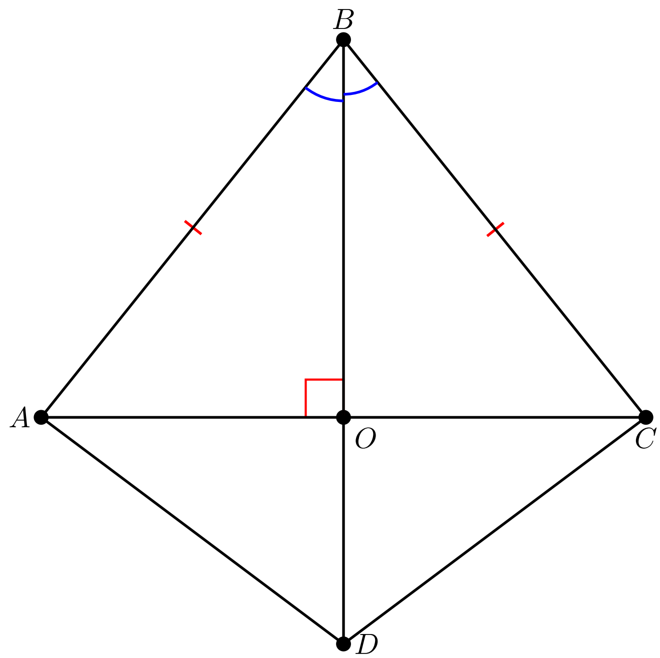
(a) Obtuse.
(b) Acute.
(c) $35^\circ 30'$, $54^\circ 30'$, $90^\circ, 90^\circ$.

□

2. *(10 pts)*
Recall that a **kite** is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are adjacent to each other.

Show that if $AB = BC$ and AC is orthogonal to BD in a convex quadrilateral $ABCD$, then $ABCD$ is a kite.

Solution. Denote by O the intersection of AC and BD. Then BO is an altitude of $\triangle ABC$. Since $\triangle ABC$ is isosceles ($AB = CB$), BO is also a bisector. Therefore, $\angle ABD = \angle CBD$. By SAS-test, ABD and CBD are congruent triangles; we obtain $AD = CD$.



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3. *(10 pts)*

Consider a triangle ABC and let AD be its median. Show that the line AD is equidistant from B and C.

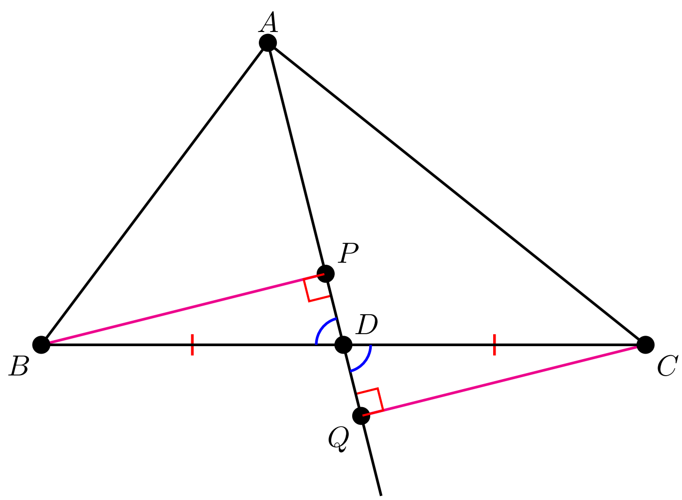
Solution. The distance from a point to a line is the length of the perpendicular dropped from the point to the line.

Let BP and CQ be the perpendiculars dropped from B and C onto the line AD. We need to show that $BP = CQ$.

We have:

- $BD = CD$ because AD is a median;
- $\angle BDP = \angle CDQ$ as vertical angles.

Therefore, the right triangles BPD and CQD are congruent; we obtain $BP = CQ$.



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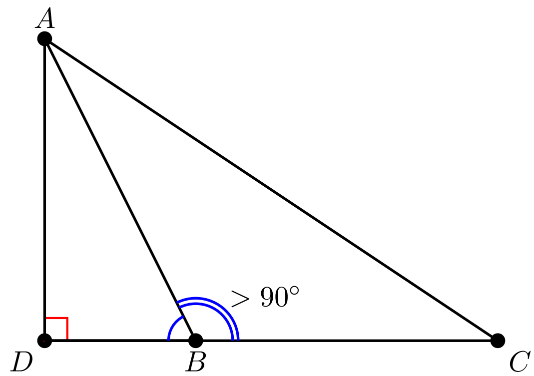
4. *(3+7 pts)*

(a) Let ABC be an acute triangle and let AD be its altitude. Show that AD is inside ABC.

(b) Suppose that $A'B'C'$ is another acute triangle and $A'D'$ is an altitude of $\triangle A'B'C'$. Show that if $AB = A'B'$, $\angle CAB = \angle C'A'B'$, and $AD = A'D'$, then ABC and $A'B'C'$ are congruent triangles.

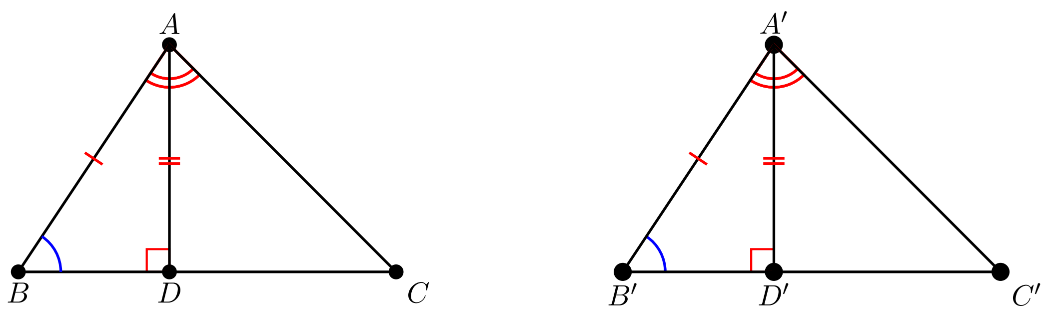
Solution. **(a)**. Suppose that AD is outside $\triangle ABC$. We assume that D is on the left of B at it shown on the figure below. The case when D is on the right of C is similar.

Since $\triangle ADB$ is right, we have $\angle DBA < 90°$. Thus $\angle ABC > 90°$ as a supplementary angle of $\angle DBA$. This is a contradiction to the assumption that $\triangle ABC$ is acute.



(b) Since $AB = A'B'$ and $AD = A'D'$, the right triangles ABD and $A'B'D'$ are congruent. As a consequence, $\angle ABC = \angle A'B'C'$.

The triangles ABC and $A'B'C'$ are congruent by ASA-test: $\angle BAC = \angle B'A'C'$, $AB = A'B'$, $\angle ABC = \angle A'B'C'$.



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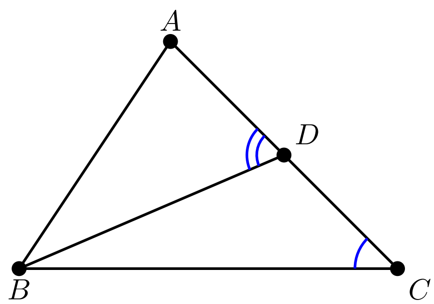
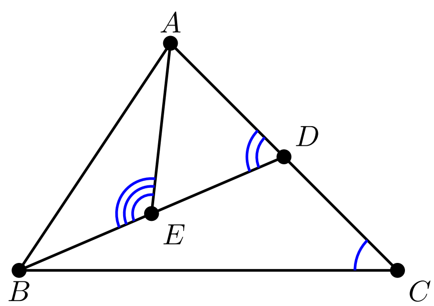
5. *(3+3+3+3 pts)*
Consider a triangle ABC.
(a) Suppose D is a point on AC strictly between A and C. Show that $\angle ADB > \angle ACB$.

Consider now a point E strictly inside $\triangle ABC$.
(b) Show that $\angle AEB > \angle ACB$.

Solution.
(a) Since $\angle ADB$ is an external angle of $\triangle DCB$, we have $\angle ADB > \angle ACB$.



(b) Let us extend BE towards AC; we denote by D the intersection of the lines BE and AC.
Consider $\triangle ABD$. By **(a)** we have $\angle BEA > \angle BDA$.
Now consider $\triangle ABC$. Again, by **(a)** we have $\angle BDA > \angle BCA$.
Therefore, $\angle BEA > \angle BCA$.



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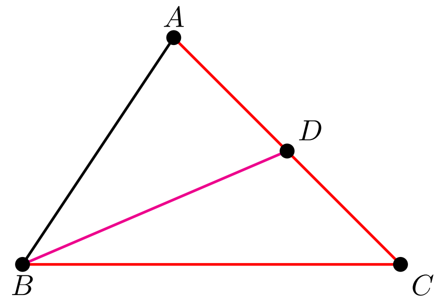
(c) Show that $AE + BE < AC + BC$.
(d) Show that $AE + BE + CE < AB + BC + CA$.

Solution.

(c) Let us first prove that $AD + BD < AC + BC$ for a point D on the side AC.

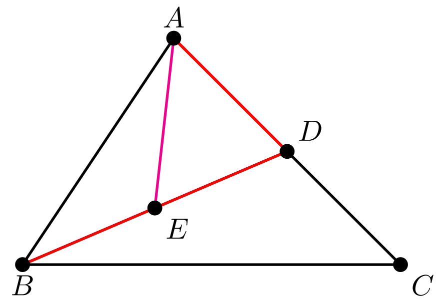
The inequality $AD + BD < AC + BC$ is equivalent to $BD < DC + BC$ – this is the triangle inequality.



Let us now consider E strictly inside $\triangle ABC$. Extend BE towards AC; we denote by D the intersection of the lines BE and AC.

Consider $\triangle ABD$. By what we just proved, $AE + BE < AD + BD$.

Now consider $\triangle ABC$. Again, we have $AD + BD < AC + BC$.
Therefore, $AE + BE < AC + BC$.



(d) It follows from **(c)** that

$$AE + BE < AC + BC,$$
$$AE + CE < AB + CB,$$
$$BE + CE < BA + CA;$$

taking the sum we obtain:

$$2(AE + BE + CE) < 2(AB + BC + AC),$$

or:

$$AE + BE + CE < AB + BC + AC.$$

□