

Sample Final Exam
MAT 515, Fall 2019
December 11, 2019
Stony Brook University

Name: (please print)	ID #:
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	1	2	3	4	5	6	7	8	9	Total
	10pts	10pts	10pts	10pts	12pts	12pts	12pts	12pts	12pts	100pts
<i>Grade</i>										

No notes or books.

You must provide explanation, not just the answer (unless otherwise is stated).

Answers without justification will get only partial credit.

Please cross out anything that is not a part of your solution — e.g., some preliminary computations that you didn't need.

Instructor: Dzmitry Dudko

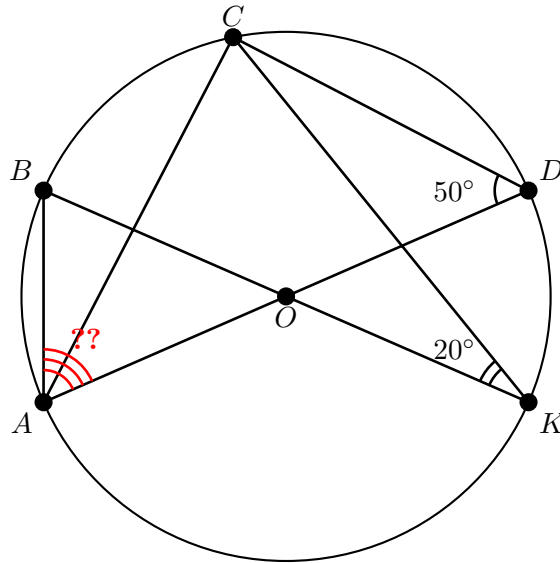
1. (10 pts)

Indicate whether each of the statements below is True (**T**) or False (**F**).
No explanation is required.

- (a) There is a triangle ABC such that $AB = 4$, $AC = 6$, and $BC = 9$.
- (b) In a triangle ABC , the external angle of A is equal to the sum of the internal angles of B and C .
- (c) Suppose $AB > BC$ in a triangle ABC . Then $\angle C > \angle A$.
- (d) There is a right triangle ABC such that $\angle A = 60^\circ$ and $\angle B = 50^\circ$.
- (e) Let AB and CD be two chords of a circle with center O . If $AB > CD$, then the distance between O and AB is greater than the distance between O and CD .
- (f) The sum of opposite angles in a parallelogram is 180° .
- (g) There is a triangle that has 6 axes of symmetry.
- (h) A square is an inscribed and circumscribed quadrilateral.
- (i) If $ABCD$ is a rectangle, then the distance between A and B is equal to the distance between AD and BC .
- (j) A median always splits a triangle into two similar triangles.

2. (10 pts)

On the figure below $\angle BKC = 20^\circ$, $\angle ADC = 50^\circ$ and BK, AD are diameters. Compute $\angle BAD$.



3. (10 pts)

Let $ABCD$ be a trapezoid with parallel bases AB and CD . Prove that the internal angle bisectors of the angles adjacent to the lateral side BC are perpendicular to each other.

4. (10 pts)

Let $ABCD$ be a circumscribed trapezoid with perimeter 4 (i.e., $AB+BC+CD+DA = 4$).
What is the length of the midline of $ABCD$?

5. (12 pts)

Consider a triangle ABC . Suppose that M is the intersection of the medians of $\triangle ABC$ and N is the intersection of the altitudes of $\triangle ABC$. Show that if $N = M$, then $\triangle ABC$ is equilateral.

6. (12 pts)

Four houses A, B, C, D form vertices of a square. The residents would like to dig a well at a point W such that the sum of distances $AW + BW + CW + DW$ from all the houses to the well is the smallest possible. Where should they dig the well?

7. (12 pts)

Construct a trapezoid $ABCD$ with bases $BC < AD$, given AB, BC, CD, DA .

8. (12 pts)

Let PA and PB be two tangents from point P to a given circle such that points A and B are the points of tangency. Construct a circle tangent to the given circle and to both lines PA and PB .

9. (6+6 pts)

Consider a triangle ABC and let D be a point on the side AC . Suppose $AB = 20$, $AD = 16$, $BD = 12$, $DC = 9$.

- (a) Prove that $\triangle ABD$ is right.
- (b) Compute BC .