MAT 515: Geometry for Teachers Problem Set 4

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Problem 1. (5 points)

Consider an equilateral triangle ABC, and let C_1, A_1, B_1 be the middle points of the sides AB, BC, CA respectively. Prove that $\triangle AB_1C_1$, $\triangle A_1BC_1$, $\triangle A_1B_1C$, $\triangle A_1B_1C_1$ are congruent triangles.

Problem 2. (5 points) Let ABCD be a convex quadrilateral. Show that

$$AC + BD > \frac{AB + BC + CD + DA}{2}.$$

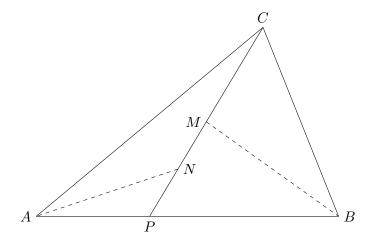
Hint: the diagonals AC, BD decompose ABCD into 4 triangles.

Problem 3. (5 points)

Show that a triangle is isosceles if two of its altitudes are congruent.

Problem 4. (5 points)

Consider a triangle ABC. Suppose P is a point on AB strictly between A and B. Let us drop perpendiculars AN and BM to the line CP. Can it happen that both M and N are strictly between C and P? Explain your answer.



Problem 5. (5 points)

Prove that the sum of segments connecting a point inside a triangle with its vertices is larger than its semiperimeter.

Hint: Decompose the triangle into 6 right triangles.

Due Date: Wednesday October 2.

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