MAT 515: Geometry for Teachers

Problem Set 12

Stony Brook University Fall 2019
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Problem 1. (6 points) Let $ABCD$ be a trapezoid where $BC < AD$ are its bases. Prove that if $\angle BAD = \angle CDA$, then $AB = DC$, i.e., $ABCD$ is an isosceles trapezoid.

Recall that the three (possibly extended) altitudes intersect in a single point, called the **orthocenter** of the triangle.

Problem 2. (5+1 points)
Let H be the orthocenter of $\triangle ABC$. Prove that C is the orthocenter of $\triangle ABH$. When do C and H coincide?

Problem 3. (6 points)
Using a compass and a straightedge, construct a triangle ABC given AC and the lengths of two medians belonging to the vertices A and C.

Hint. Let AA_1 and CC_1 be the medians of $\triangle ABC$ belonging to the vertices A and C. Construct first $\frac{2}{3}AA_1$ and $\frac{2}{3}CC_1$. Use Problem 3 (b) of Midterm 2 to trisect a given line segment.

Problem 4. (5+1 points)
Let $ABCD$ be a rhombus, and let O be the intersection of the diagonals AC and BD. Drop the perpendiculars OP, OQ, OT, OS onto the sides AB, BC, CD, DA. Prove that $\triangle AOP, \triangle OBP, \triangle OBQ, \triangle COQ, \triangle COT, \triangle ODT, \triangle ODS, \triangle AOS$ are similar.
Is $OP = OQ = OT = OS$?

Problem 5. (*Bonus problem*, 5 points)
Using a compass and a straightedge, inscribe a circle into a given rhombus.

Due Date: Wednesday December 4.