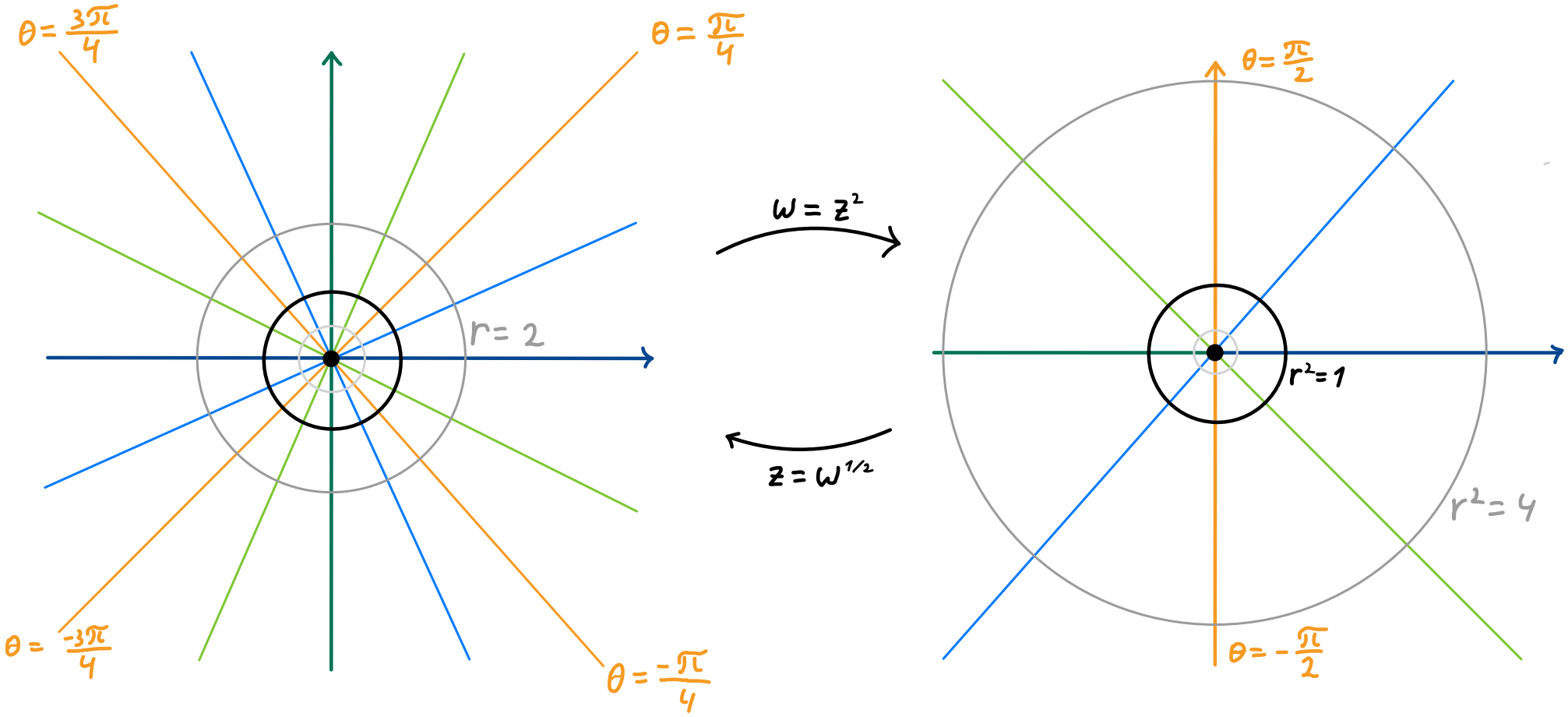
Visualization of $w = z^2$

Since $w = z^2 = [r(\cos\theta + i\sin\theta)]^2 = r^2(\cos(2\theta) + i\sin(2\theta))$, we have:



Visualization of $w = z^2$

Since $z^2 = (x+iy)^2 = \underbrace{(x^2-y^2)}_{a} + \underbrace{(2xy)}_{b}i = a + ib = w$,

the pre-images of the vertical lines $\operatorname{Re} w = a \neq 0$ are the hyperbolas $x^2 - y^2 = a$

the pre-images of the horizontal lines $\operatorname{Im} w = b \neq 0$ are the hyperbolas $2xy = b$