MAT 211: Linear Algebra
Problem Set 8

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If A, B are $n \times n$ matrices, then

$$\det(AB) = \det(A)\det(B).$$

Problem 1. (5 points) Consider $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & 0 \\ 0 & f \end{bmatrix}$. Compute AB, $\det(A)$, $\det(B)$, $\det(AB)$. Verify that indeed

$$\det(A)\det(B) = \det(AB).$$

Problem 2. (5 points) Find all the eigenvalues of $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$. Give bases for each of the corresponding eigenspaces.

Problem 3. (5 points) Find all the eigenvalues of $A = \begin{bmatrix} 2 & 4 \\ 6 & 0 \end{bmatrix}$. Give bases for each of the corresponding eigenspaces.

Problem 4. (2+3 points) Show that $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ are eigenvectors of

$$A = \begin{bmatrix} -3 & 6 \\ -2 & 5 \end{bmatrix}.$$

Compute $A^3 \begin{bmatrix} 2 \\ 0 \end{bmatrix} = A^3 \left(\begin{bmatrix} 3 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$.

Problem 5. (5 points)
Suppose that v and w are eigenvectors of a matrix A corresponding to eigenvalues 2 and 3; i.e. $Av = 2v$ and $Aw = 3w$. Prove that v and w are linearly independent.
Hint: consider $c_1 v + c_2 w = 0$ and multiply this equation by A.

Problem 6. (3 points) Find t such that

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ -1 \end{bmatrix} - t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

is an orthogonal basis for $\mathbb{R}^2$.

Due Date: Tuesday, Nov 30.