MAT 211: Linear Algebra

Problem Set 7

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Problem 1. (2 points) Compute the standard matrix of the linear transformation

$$P\begin{bmatrix} x \\ y \end{bmatrix} = (2x+7y)\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Problem 2. (4 +4 +4 points)
Recall that

- a vector u is **unit** if it has length 1; i.e. if $||u|| = \sqrt{u \cdot u} = 1$;
- if v is a non-zero vector, then $\frac{1}{||v||}v$ is a unit vector parallel to v.

Consider a unit vector $d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$. Denote by ℓ the line passing through the origin whose direction vector is d. In other words,

$$\begin{bmatrix} x \\ y \end{bmatrix} = t\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

is the vector form of the equation of ℓ. The linear transformation

$$P_\ell(v) = (v \cdot d)d = (v_1 d_1 + v_2 d_2)\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

is a projection of the plane $\mathbb{R}^2$ onto the line ℓ, where $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$.

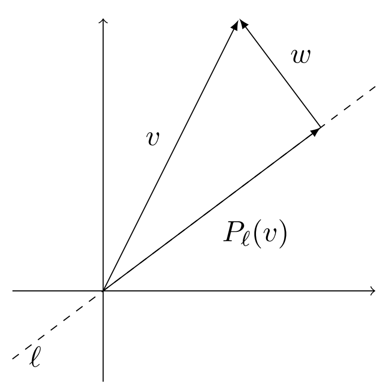
1) Prove that

$$P_\ell(v) = (v \cdot d)d = (v_1 d_1 + v_2 d_2)\begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

is the orthogonal projection of $\mathbb{R}^2$ onto the line ℓ by showing that the vector

$$w = v - P_\ell(v)$$

is orthogonal to d.



2) Suppose that $d = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$. Check that d is a unit vector. Compute the standard matrix of the projection P_ℓ. Compute $P_\ell\begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

3) Suppose that $d = \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$. Check that d is a unit vector. Compute the standard matrix of the projection P_ℓ. Compute $P_\ell \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Due Date: Thursday, Nov 4.