Practice for sections 4.3, 4.5, 4.6 and 4.7
MAT 131 – Fall 2009

Here are a few practice problems, related to sections 4.3, 4.5, 4.6 and 4.7. For practice on earlier sections, see the earlier collections of practice problems. The homework problems, that you are doing at the present time, provide a lot of practice for for Chapter 5. Since there is already a lot of homework on Chapter 5, and since the exam is so soon, I won't burden you with additional practice problems for Chapter 5. (Section 4.8 is about antiderivatives, and you are already getting a lot of practice in finding those, in sections 5.3 and 5.5.)

Important Note: These problems are samples of what you might see on a MAT 131 exam. The actual exam can and will have problems of types not to be found here. Doing practice problems before the exam is no substitute for attending and participating in class, regularly doing homework, and reviewing the course material and the assigned homework before the exam.

At the end you will find answers, hints and brief solution sketches. On the exam, you must show all your work, so a numerical answer, even if correct, would not be acceptable just by itself.

1. Page 280, #23, 33

2. Find

   (a) $\lim_{x\to 0} \frac{e^x - 1 - x - \frac{x^2}{2}}{x^3}$

   (b) $\lim_{x\to 0^+} x^2 (\ln x)^{10}$

   (c) $\lim_{x\to 0^+} x^{1/\ln x}$

   (d) $\lim_{x\to 0^+} x^{(x^{1/5})}$

3. Which isosceles triangles, inscribed in a circle of radius 1, have the largest area?

4. Use Newton's method with initial approximation $x_1 = 1$ to find x_2, the second approximation to the root of $x^3 + 2x - 4 = 0$.

5. Chapter 5: memorize the table on page 358, and do the homework!!!!

Answers, Hints and Brief Solution Sketches

1. Answers are in the back of the book.

2. (a) Answer: 1/6.
Hint: Apply L'Hospital's Rule 3 times.
(b) Answer: 0.
Hint: Let $y = x^{1/5} \ln x$. Show that y approaches 0 as $x \to 0$, by writing $y = \ln x / x^{-1/5}$ and using L'Hospital's rule. Then note that $\lim_{x \to 0^+} x^2 (\ln x)^{10} = \lim_{x \to 0^+} y^{10} = 0$.
(c) Answer: e.
Solution: Let $z = x^{1/\ln x}$. Then $\ln z = (1/\ln x)(\ln x) = 1$, so $z = e^1 = e$ for all x.
(d) Answer: 1.
Solution: Let $z = \lim_{x \to 0^+} x^{(x^{1/5})}$. Then $\ln z = x^{1/5} \ln x = y$ (same y as in part (b)). As we saw in part (b), $\lim_{x \to 0^+} y = 0$. So, as $x \to 0$, $z = e^y \to e^0 = 1$.

3. Answer: equilateral triangles.
Solution: Say the circle has center 0. Say the vertices of the triangle are A, B, C, with $AB = AC$. Suppose that $\angle AOB = \theta$, so that (since ΔAOB is congruent to ΔAOC), $\angle AOC = \theta$ also, and therefore $\angle BOC = 2\pi - 2\theta$. By problem 46(a) on page A26, the area of ΔABC, which is the sum of the areas of ΔAOB, ΔAOC and ΔBOC, is

$$A(\theta) = \frac{1}{2}\sin\theta + \frac{1}{2}\sin\theta + \frac{1}{2}\sin(2\pi - 2\theta) = \sin\theta - \frac{1}{2}\sin 2\theta.$$

It is enough to find out where the maximum of $A(\theta)$ is on $[0, \pi]$. At $\theta = 0$ or $\theta = \pi$, $A(\theta) = 0$, so the max must occur at a critical point in $(0, \pi)$. We calculate

$$A'(\theta) = \cos\theta - \cos 2\theta.$$

We have $0 \le \theta \le \pi$. At a critical point, $\cos\theta = \cos 2\theta$. Now, if α, β are two angles with $0 < \alpha, \beta < 2\pi$, then the only way that they can have the same cosine is if $\alpha = \beta$ or if $\alpha + \beta = 2\pi$. So we must have $\theta + 2\theta = 2\pi$, so that $\theta = 2\pi/3$ and the triangle is equilateral. (You could also show that $\theta = 2\pi/3$ by using the double-angle formula for cosine.)

4. Answer: $x_2 = 6/5$.
Solution: Note that $f'(x) = 3x^2 + 2$. Thus $x_2 = x_1 - [f(x_1)/f'(x_1)] = 1 - [(-1)/5] = 6/5$.