Practice for Midterm 2 – MAT 131 – Fall 2009

Important Note: These problems are samples of what you might see on a MAT 131 midterm. The actual midterm can and will have problems of types not to be found here. Doing practice problems before a midterm is no substitute for attending and participating in class, regularly doing homework, and reviewing the course material and the assigned homework before the exam.

At the end you will find answers, hints and brief solution sketches. On the midterms, you must show all your work, so a numerical answer, even if correct, would not be acceptable just by itself.

1. In each part, calculate y'.

   (a)
$$y = (x^4 - 5x^3 + 7)^6$$

   (b)
$$y = \sqrt{x^3} + \frac{6}{\sqrt{x^5}}.$$

   (c)
$$y = 7x^3\sqrt{x^2 + x}.$$

   (d)
$$y = \sin^{-1}(e^{2x}).$$

   (e)
$$y = \log_7(3^x + \tan^{-1} x).$$

   (f)
$$y = \cos\left(\frac{\sin x}{e^x - 1}\right)$$

   (g)
$$y = \frac{(x^4 + 1)^{5/2}\sin^2 x}{(x^2 + x + 1)^4}$$

   (h)
$$y = x^x$$

   (i)
$$\sin(xy) = x^2 - y^3$$

2. Page 215, #35. Also: what is the equation of the tangent line at the point where $x = 0$?

3. Page 249, #65.

4. Let $f(x) = x^3 + x + 1$, defined on the whole real line. Show that f is one-to-one. Let g be the inverse function to f. Assume that g is differentiable (it is!). If $y = g(x)$, find y' in terms of x and y.

5. Use linearization to find an approximation to $\arctan 1.03$. Is your approximation an overestimate or an underestimate?

6. Page 237-238, #7.

7. Pages 260-261, #17, 29, 31.

8. Page 269, #41, 47, 49.

Answers, Hints and Brief Solution Sketches

1. (a) Answer: $y' = 6(x^4 - 5x^3 + 7)^5(4x^3 - 15x^2)$.
Hint: use the chain rule.
Similar problem in the book: Example 3, page 200.

(b) Answer: $y' = \frac{3}{2}x^{1/2} - 15x^{-7/2}$.
Hint: First write $y = x^{3/2} + 6x^{-5/2}$.
Similar problem in the book: Example 2, page 176.

(c)Answer: $y' = 21x^2(x^2 + x)^{1/2} + \frac{7}{2}x^3(x^2 + x)^{-1/2}(2x + 1)$.
Hint: First write $\sqrt{x^2 + x} = (x^2 + x)^{1/2}$, then use the product rule and the chain rule.
Similar problem in the book: Example 6, page 201.

(d) Answer:

$$y' = \left[\frac{1}{\sqrt{1 - e^{4x}}}\right](2e^{2x}).$$

Hint: Use the chain rule.
Similar problem in the book: Example 5(b), page 220.

(e) Answer:

$$y' = \left[\frac{1}{(\ln 7)(3^x + \tan^{-1}(x))}\right]\left(3^x \ln 3 + \frac{1}{1 + x^2}\right).$$

Hint: use the chain rule.
Similar problem in the book: Example 4, page 222.

(f) Answer:

$$y' = -\sin\left(\frac{\sin x}{e^x - 1}\right)\frac{\cos x(e^x - 1) - (\sin x)e^x}{(e^x - 1)^2}.$$

Hint: use the chain rule, then the quotient rule.
Similar problem in the book: Example 5, page 201.

(g) Answer:

$$y' = \frac{(x^4 + 1)^{5/2}\sin^2 x}{(x^2 + x + 1)^4}\left(\frac{10x^3}{x^4 + 1} + \frac{2\cos x}{\sin x} - \frac{4(2x + 1)}{x^2 + x + 1}\right).$$

Hint: logarithmic differentiation.
Similar problem in the book: Example 7, page 223.

(h) Answer: $y = x^x(\ln x + 1)$.
Hint: logarithmic differentiation.
Similar problem in the book: Example 8, page 225.

(i) Answer:

$$y' = \frac{2x - y\cos(xy)}{x\cos(xy) + 3y^2}.$$

Hint: implicit differentiation. The derivative of $\sin(xy)$ with respect to x is $\cos(xy)(xy)'$; next use the product rule.
Similar problem in the book: Example 3, page 213.

2. Answer: $y'' = 1/e^2$ there; tangent line is $y - 1 = -x/e$.
Hint: First note that at the point where $x = 0$, we have $e^y = e$, so $y = 1$. Differentiate the equation $xy + e^y = e$ once to find $y + xy' + e^y y' = 0$, and again to find $y' + y' + xy'' + e^y(y')^2 + e^y y'' = 0$. At $(0, 1)$, the first equation tells us that $y' = -1/e$; using this in the second equation, we find that $y'' = 1/e^2$ at $(0, 1)$.

3. Answer: in the back of the book.
Hint: use implicit differentiation to find the derivative. Use it together with the equation for the ellipse, $x^2 + 2y^2 = 1$. Similar problem in the book: Example 2, page 212.

4. $f'(x) = 3x^2 + 1$, which is always positive. So f is increasing, hence it is one-to-one. Since $f(g(x)) = x$ for all x, $f(y) = x$. Differentiate implictly to find $y' = \frac{1}{3y^2+1}$.
Similar example in the book: derivation of the derivative of $\sin^{-1}$, bottom of page 217.

5. Linearization says that

$$f(x) \sim f(a) + f'(a)(x - a)$$

for f differentiable and x near a. Let $f(x) = \arctan x$, $a = 1$, $x = 1.03$. We find

$$arctan 1.03 \sim \arctan 1 + \frac{1}{2}(.03) = \frac{\pi}{4} + .015.$$

It's an overestimate because the tangent line $y = f(a) + f'(a)(x - a)$ lies above the graph of arctan near the ppint $(1, \pi/4)$ on the graph.
Similar example in the book: Example 2, page 241.

The answers to the rest of the problems are in the back of the book.