Practice for Midterm 1 – MAT 131 – Fall 2009

Important Note: These problems are samples of what you might see on a MAT 131 midterm. The actual midterm can and will have problems of types not to be found here. Doing practice problems before a midterm is no substitute for attending and participating in class, regularly doing homework, and reviewing the course material and the assigned homework before the exam.

At the end you will find answers, hints and brief solution sketches. On the midterms, you must show all your work, so a numerical answer, even if correct, would not be acceptable just by itself.

1. For the function
$$f(x) = \frac{\sqrt{x^2+1} - \sqrt{x+1}}{x},$$
find the domain and determine if the function is even, odd or neither. Find the location and type of all discontinuities, and determine all horizontal and vertical asymptotes.

2. Find the limits.

(a)
$$\lim_{x\to 2} \frac{x^2 - 3x + 2}{x - 2}$$

(b)
$$\lim_{x\to 2} \frac{x^2 - 3x + 2}{x^2 - 4}$$

(c)
$$\lim_{h\to 0} \frac{(2+h)^2 - 4}{h}$$

(d)
$$\lim_{t\to 1} |t^2 - 1|$$

(e)
$$\lim_{x\to 2} \frac{3x}{x - 2}$$

(f)
$$\lim_{x\to 2} \frac{3x}{|x - 2|}$$

(g)

$$\lim_{x\to\infty} \frac{3x^4 - 5x^2 + 2}{2x^4 - x^3 + 7}$$

(h)

$$\lim_{x\to-\infty} \frac{3x^4 - 5x^2 + 2}{2x^3 - x^2 + 7}$$

(i)

$$\lim_{x\to\infty} \frac{\cos x}{x}$$

(j)

$$\lim_{x\to\infty} \frac{x + \cos x}{x}$$

(k)

$$\lim_{x\to\infty} x \cos x$$

(l)

$$\lim_{x\to(\pi/2)^-} e^{-\tan x}$$

(m)

$$\lim_{x\to(\pi/2)^+} e^{-\tan x}$$

(n)

$$\lim_{x\to 0} \frac{\sin x}{\tan x}$$

3. Find the derivative of f, directly from the definition.
   Also find the equation of the tangent line at the given point a.

   (a) $f(x) = 2x^2 - 3x$.
   $a = -1$.

   (b) $f(x) = \sqrt{x+2}$
   $a = 2$.

   (c) $f(x) = 1/\sqrt{x}$
   $a = 9$.

4. page 81, # 13, #15

5. page 81, #14

6. page 156, #13

7. page 163, #21

Answers, Hints and Brief Solution Sketches

1. For the function to be defined at x, we need $x+1 \geq 0$ and $x \neq 0$, so the domain is the set of points in $[-1, \infty)$ except for 0.

   The function is not even or odd, since

   $$f(-x) = \frac{\sqrt{x^2+1} - \sqrt{-x+1}}{-x},$$

   which does not equal $f(x)$ or $-f(x)$.

   The function is continuous at each point in its domain, since the numerator and denominator are, by the box on page 118. At -1 it is continuous from the right (it is not defined to the left of -1).

   At 0 the limit exists and equals $-1/2$ (similar problem: page 106, example 6). So there is a removable discontinuity there, and there are no places where there could be vertical asymptotes.

   As for horizontal asymptotes, $\lim_{x\to\infty} f(x) = 1$, so the line $y = 1$ is the only horizontal asymptote. (To see that $\lim_{x\to\infty} f(x) = 1$, divide numerator and denominator by x, and note that $\sqrt{x^2+1}/x = \sqrt{1+1/x^2}$ and $\sqrt{x+1}/x = \sqrt{1/x+1/x^2}$.)

2. (a) Answer: 1
   Hint: Factor the numerator.
   Similar problem in the book: Example 3, page 107.

   (b) Answer: 1/4
   Hint: Factor both the numerator and the denominator.
   Similar problem in the book: Example 3, page 107.

   (c) Answer: 4
   Hint: Expand out $(2+h)^2$.
   Similar problem in the book: Example 5, page 106.

   (d) Answer: 0
   Let $x = t^2 - 1$. As $t \to 1$, $x \to 0$, so $|t^2-1| = |x| \to 0$ by Example 7, page 109.

   (e) Answer: DNE (does not exist)
   The limit from the right is ∞ and from the left is $-\infty$.
   Similar problem in the book: Example 1, page 126.

   (f) Answer: ∞

The limit from the left or the right is ∞.
Similar problem in the book: Example 1, page 126.

(g) Answer: 3/2
Hint: Divide the numerator and the denominator by x^4.
Similar problem in the book: Example 5, page 130.

(h) Answer: $-\infty$
Hint: Divide the numerator and the denominator by x^3.
Similar problem in the book: Example 5, page 130.

(i) Answer: 0
Hint: Use the squeeze rule, noting that, for $x > 0$,

$$\frac{-1}{x} \leq \frac{\cos x}{x} \leq \frac{1}{x}.$$

Similar problem in the book: Example 10, page 110.

(j) Answer: 1
Hint: Divide the numerator and the denominator by x, and use the answer to problem (i).
Similar problem in the book: Example 5, page 130.

(k) Answer: DNE
At odd multiples x of π, the function alternately equals x or $-x$. So as $x \to \infty$, it takes on arbitrarily large positive values and arbitrarily large negative values.
Similar problem in the book: Example 8, page 131.

(l) Answer: 0
Let $t = \tan x$. As $x \to (\pi/2)^-$, $t \to \infty$, so $e^{-\tan x} = e^{-t} \to 0$.
Similar problem in the book: Example 7, page 131.

(m) Answer: ∞
Let $t = \tan x$. As $x \to (\pi/2)^-$, $t \to -\infty$, so $e^{-\tan x} = e^{-t} \to \infty$.
Similar problem in the book: Example 7, page 131.

(n) Answer: 1
$\tan x = \sin x / \cos x$, so this is just $\lim_{x \to 0} \cos x$.

3. (a) Derivative: $4x - 3$.
Similar problem: Example 4, page 138. Go through the whole procedure, as on page 139, don't just write down the answer!!!
$f(-1) = 5$, $f'(-1) = -7$, so tangent line when $a = -1$ has equation $y - 5 = -7(x + 1)$.

(b) Derivative: $1/(2\sqrt{x+2})$
Similar problem: Example 4, page 149. Go through the whole procedure, as on page 149, don't just write down the answer!!!
$f(2) = 2$, $f'(2) = 1/4$, so tangent line when $a = 2$ has equation $y - 2 = (1/4)(x - 2)$.

(c) Derivative: $-1/(2x^{3/2})$
Similar problem: Example 4, page 149. Go through the whole procedure, as on page 149, don't just write down the answer!!!
Hint: write

$$[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}]/h = [\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h}\,\sqrt{x}}]/h = \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h}\,\sqrt{x}\,h}$$

then multiply and divide by

$$\sqrt{x} + \sqrt{x+h}$$

$f(9) = 1/3$, $f'(9) = -1/54$, so tangent line when $a = 9$ has equation
$y - (1/3) = -(1/54)(x - 9)$.

4. See solutions in the back of the book.

5.
$$f^{-1}(x) = \frac{1-x}{2x-1}.$$

If $y = f(x)$, then $y = (x+1)/(2x+1)$. To get $f^{-1}(x)$, interchange x and y and solve for y in terms of x.
Similar problem in the book: Example 4, page 64.

6. See solutions in the back of the book.

7. See solutions in the back of the book.