HOMEWORK 2

PROBLEMS FROM THE TEXTBOOK

§18, Exercises 2, 5, 8, 9(ab), 13

OTHER PROBLEMS

1. Let $X$ be a Hausdorff topological space. Prove that the limit of every convergent sequence in $X$ is unique.

2. Let $X$ be the set of all functions $f : \mathbb{N} \to \mathbb{R}$; such a function is of course the same thing as the sequence of real numbers $f(0), f(1), \ldots$.
   (a) For every $g \in X$ and every sequence of positive real numbers $\varepsilon_0, \varepsilon_1, \ldots$, we can form the following subset of $X$:
       \[ \{ f \in X \mid |f(n) - g(n)| < \varepsilon_n \text{ for every } n = 0, 1, \ldots \} \]
       Prove that the collection of all such subsets is a basis for a topology on $X$.
       Show that this topology does not satisfy the first countability axiom.
   (b) Find an example of a subset of $X$ that is sequentially closed but not closed.

Due on Monday, September 8.