MAT132, Paper Homework 8

due in recitation on 12/5, 12/6, or 12/7

1. A small country has \$10 billion in paper currency in circulation, and each day, \$20 million comes into the country's banks. The government decides to introduce new currency by having the banks replace old bills with new ones whenever old currency comes into the banks.

   (a) Let $x(t)$ denote the amount of new currency in circulation at time t. Formulate a mathematical model, in the form of an initial-value problem, that represents the "flow" of the new currency into circulation.

   (b) Solve this initial-value problem.

   (c) How long will it take for the new bills to account for 90% of the currency in circulation?

2. Let c be a positive number. A differential equation of the form

$$\frac{dy}{dt} = ky^{1+c}$$

   where k is a positive constant, is called a *doomsday equation*.

   (a) Determine the solution that satisfies the initial condition $y(0) = y_0$.

   (b) Show that there is a finite time $t = T$ (doomsday) such that $\lim_{t \to T^-} y(t) = \infty$.