MAT132, Paper Homework 6

due in recitation on 11/14, 11/15, or 11/16

1. Suppose that the radius of convergence of the power series $\sum c_n x^n$ is R. What is the radius of convergence of the power series $\sum c_n x^{2n}$?

2. In this problem, we are going to express π as an infinite series.

   (a) By completing the square, show that

   $$\int_0^{1/2} \frac{dx}{x^2 - x + 1} = \frac{\pi}{3\sqrt{3}}.$$

   (b) Express the functions $1/(x^3+1)$ and $(x+1)/(x^3+1)$ as power series.

   (c) With the help of the factorization $x^3 + 1 = (x+1)(x^2 - x + 1)$, rewrite the integral in part (a). Then use the power series from part (b) to prove the following formula:

   $$\pi = \frac{3\sqrt{3}}{4} \sum_{n=0}^{\infty} \frac{(-1)^n}{8^n} \left(\frac{2}{3n+1} + \frac{1}{3n+2} \right).$$

   Note: This series converges fast enough to be useful for computations. If you are interested, see how many digits of π you can get by adding the first few terms of the series!