1. In this problem, we consider the following curve, known as a catenoid.

It is the shape that a rope makes when it is hanging down from two points. Mathematically, the catenoid is the graph of the function \( f(x) = \cosh x \). In case you don’t know,

\[
\cosh x = \frac{e^x + e^{-x}}{2} \quad \text{and} \quad \sinh x = \frac{e^x - e^{-x}}{2}
\]

are the *hyperbolic cosine* and *hyperbolic sine* functions (pronounced *cosh* and *synch*).

(a) Let \( \ell \) be the arc length of the catenoid from the point \((0, 1)\) to the point \((a, \cosh a)\). Write down an integral that represents \( \ell \).

(b) Evaluate the integral.

(c) Prove the identity \( \cosh^2 x - \sinh^2 x = 1 \), and use it to show that \( \ell = \sinh a \).