SOLUTIONS TO MIDTERM I FOR MAT 132 (SPRING 2010)

Instructions: Print your name, ID number and section number below.

Do each of the following 6 problems in the spaces provided. Show some work or give an explanation where you are asked to do so. It is not necessary to "simplify" your answers.

Please do not use calculators or any other electronic devices, notes or books during the exam time.

(1) (30 points) Compute the following indefinite integrals.

(a) $\int (x^2 + \frac{1}{(x+2)^3} - sin(3x))dx =?$

Solution: $\frac{x^3}{3} - \frac{1}{2(x+2)^2} + \frac{cos(3x)}{3} + c$

(b) (show some work) $\int cot(x)dx =?$

Solution: Subsitute $u = sin(x)$ and $du = cos(x)dx$. Then

$$\int cot(x)dx = \int \frac{cos(x)dx}{sin(x)} = \int \frac{du}{u} = ln(\mid u \mid) + c = ln(\mid sin(x) \mid) + c$$

(c) (show some work) $\int sin^3(-2x)dx =?$

Solution: Use the trig identity $sin^2(-2x) = 1 - cos^2(-2x)$ and the substitution $u = cos(-2x), du = 2sin(-2x)$. Then we have

$$\int sin^3(-2x)dx = \int (1 - cos^2(-2x))sin(-2x)dx =$$

$$\int (1-u^2)(\frac{1}{2}du) = \frac{1}{2}(u - \frac{u^3}{3}) + c = \frac{cos(-2x)}{2} - \frac{cos^3(-2x)}{6} + c$$

(2) (20 points) Let R denote the region in the plane which is bounded by the 4 curves $x = -1$, $x = 1$, $y = 0$ and $y = \frac{x+1}{(x+2)(x+4)}$.

(a) Express the area of R as a definite integral.
Solution:

$$\int_{-1}^{1} \frac{x+1}{(x+2)(x+4)} dx$$

(b) (show some work) Evaluate the definite integral of part (a).
Note that $\frac{x+1}{(x+2)(x+4)} = -\frac{1}{2(x+2)} + \frac{3}{2(x+4)}$. Thus the definite integral of part (a) is equal to

$$-\frac{1}{2}\int_{-1}^{1} \frac{1}{x+2} dx + \frac{3}{2}\int_{-1}^{1} \frac{1}{x+4} dx =$$
$$(-\frac{1}{2}ln(3) + \frac{1}{2}ln(1)) + (\frac{3}{2}ln(5) - \frac{3}{2}ln(3))$$

(3) (20 points) A particle is moving along the the x-axis; its speed at any time $t \geq 0$ is given in terms of t by the formula $\frac{ln(t+1)}{(t+1)^2}$.

(a) Express the total distance traveled by the particle during the time interval $1 \leq t \leq 3$ as a definite integral.
Solution:

$$\int_{1}^{3} \frac{ln(t+1)}{(t+1)^2} dx$$

(b) (show some work) Evaluate the definite integral of part (a).
Using integration by parts (with $u = ln(t+1)$ and $v' = \frac{1}{(t+1)^2}$) we get that

$$\int \frac{ln(t+1)}{(t+1)^2} dx = ln(t+1)(\frac{-1}{t+1}) - \int (\frac{-1}{t+1})(\frac{1}{t+1}) dt =$$
$$\frac{-ln(t+1)}{t+1} + \frac{-1}{t+1} + c$$

Thus the definite integral in part (a) is equal to

$$(\frac{-ln(4)}{4} + \frac{-1}{4}) - (\frac{-ln(2)}{2} + \frac{-1}{2})$$

(4) (30 points) For each of the following improper integrals determine whether or not it converges. If the integral converges then determine its value.

(a) (show some work)

$$\int_0^\infty e^{-x}dx$$

Solution:

$$\int_0^\infty e^{-x}dx = limit_{a\to\infty}\int_0^a e^{-x}dx = limit_{a\to\infty}(-e^a + e^0) = 0 + 1$$

This improper integral converges to 1.

(b) (show some work)

$$\int_{-1}^1 \frac{1}{(x-1)^2}dx$$

Solution:

$$\int_{-1}^1 \frac{1}{(x-1)^2}dx = limit_{a\to 1^-}\int_{-1}^a \frac{1}{(x-1)^2}dx =$$

$$limit_{a\to 1^-}(\frac{-1}{a-1}) - (\frac{-1}{-1-1}) = \infty - \frac{1}{2} = \infty$$

Thus this improper integral does not converge.