

# MAT 127

# Midterm I

February 27, 2017

8:45-10:15pm

Name: \_\_\_\_\_  
first name *first*

ID: \_\_\_\_\_

Section:      L01                      L02                      L03                      L04                      (circle yours)  
                    MWF 10-10:53am      TuTh 2:30-3:50pm      MWF 11-11:53am      TuTh 4-5:20pm

## DO NOT OPEN THIS EXAM YET

### Instructions

- (1) Fill in your name and Stony Brook ID number and circle your lecture number at the top of this cover sheet.
- (2) This exam is closed-book and closed-notes; no calculators, no phones.
- (3) Please write legibly to receive credit. Circle or box your final answers. If your solution to a problem does not fit on the page on which the problem is stated, please indicate on that page where in the exam to find (the rest of) your solution.
- (4) You may continue your solutions on additional sheets of paper provided by the proctors. If you do so, please write your name and ID number at the top of each of them and staple them to the back of the exam (stapler available); otherwise, these sheets may get lost.
- (5) Anything handed in will be graded; incorrect statements will be penalized even if they are in addition to complete and correct solutions. If you do not want something graded, please erase it or cross it out.
- (6) Leave your answers in exact form (e.g.  $\sqrt{2}$ , not  $\approx 1.4$ ) and simplify them as much as possible (e.g.  $1/2$ , not  $2/4$ ) to receive full credit.
- (7) Show your work; correct answers only will receive only partial credit (unless noted otherwise).
- (8) Be careful to avoid making grievous errors that are subject to heavy penalties.
- (9) If you need more blank paper, ask a proctor.

Out of fairness to others, please **stop working and close the exam as soon as the time is called**. A significant number of points will be taken off your exam score if you continue working after the time is called. You will be given a two-minute warning before the end.

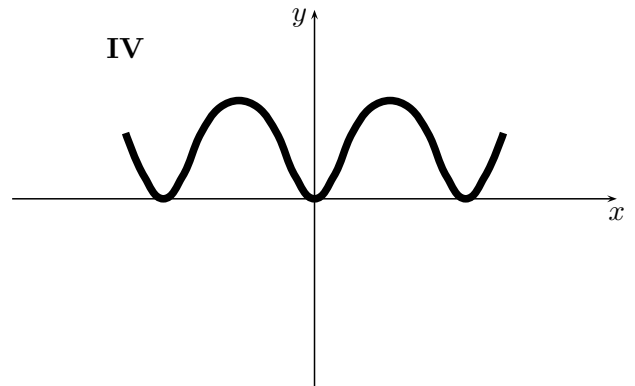
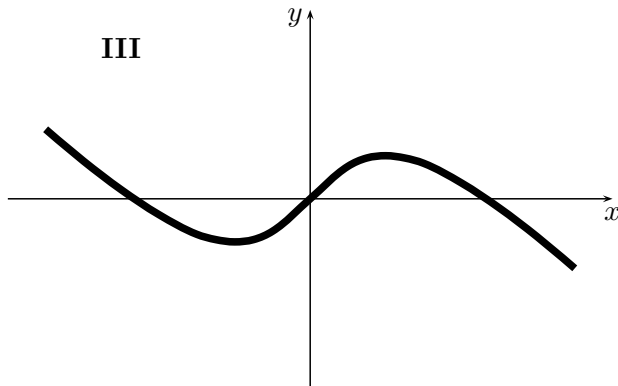
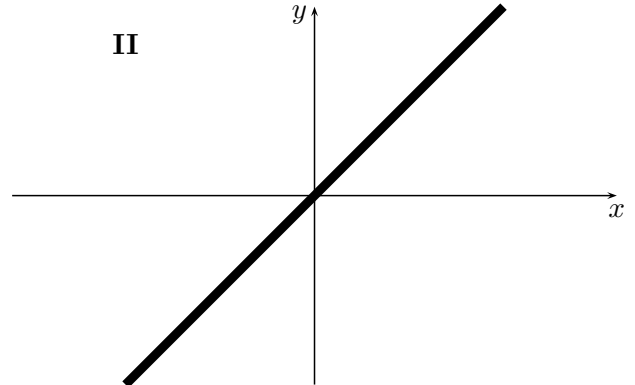
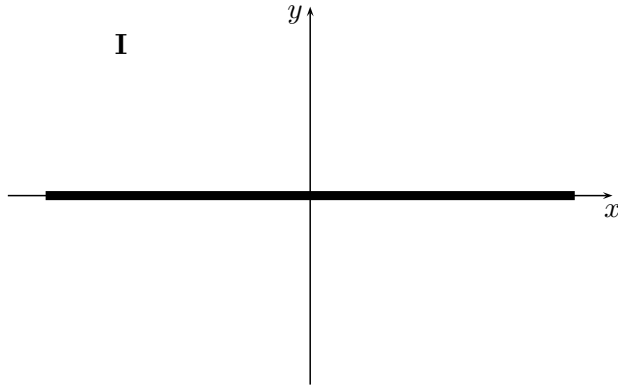
1 (10pts)	2 (10pts)	3 (15pts)	4 (20pts)	5 (20pts)	6 (25pts)	Tot (100pts)

**Problem 1 (10pts)**

Consider the four differential equations for  $y = y(x)$ :

- (a)  $y' = (\cos x)(\sin y)$     (b)  $y' = (\sin x)(\cos y)$     (c)  $y' = \cos(x-y)$     (d)  $y' = \cos(x+y)$ .

Each of the four diagrams below shows a solution curve for one of these equations:



Match each of the diagrams to the corresponding differential equation (the match is one-to-one):

diagram	I	II	III	IV
equation				

**Answer Only:** no explanation is required.

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do not write below this line or your work on this problem will be void

**grader's use only**

correct – repeats	0-	1	2	3	4
points	0	2	5	9	10

**Problem 2 (10pts)**

Suppose that the function  $f = f(x)$  is a solution of the differential equation

$$y'' + 2e^x y' + e^{2x} y = 2 \cos^2 x - \sin^2 x, \quad y = y(x),$$

and that the function  $g = g(x)$  is a solution of the differential equation

$$y'' + 2e^x y' + e^{2x} y = 2 \sin^2 x - \cos^2 x, \quad y = y(x).$$

Show that the function  $h = f + g$  is a solution of the differential equation

$$y'' + 2e^x y' + e^{2x} y = 1, \quad y = y(x).$$

*Show your work and/or explain your reasoning.*

### Problem 3 (15pts)

A radioactive substance, Strontium-90, has a half-life of 30 years. A sample of Strontium-90 initially contains 400 mg.

(a; 10pts) How much Strontium-90 remains after  $t$  years? *Show your work and/or explain your reasoning.*

(b; 5pts) How long will take for the amount of Strontium-90 to decay to 25 mg? *Show your work and/or explain your reasoning.*

**Problem 4 (20pts)**

(a; 10pts) Find the general solution of the differential equation

$$y'' + 3y' = 0, \quad y = y(x).$$

*Show your work and/or explain your reasoning.*

(b; 10pts) Find the solution of the initial-value problem

$$y'' + 3y' = 0, \quad y(0) = 0, \quad y'(0) = 3, \quad y = y(x).$$

*Show your work and/or explain your reasoning.*

### Problem 5 (20pts)

Let  $y = y(x)$  be the solution to the initial-value problem

$$y' = x + 2y, \quad y = y(x), \quad y(0) = 0.$$

(a; 15pts) Use Euler's method with  $n = 3$  steps to estimate the value of  $y(1)$ . *Show your steps clearly and use simple fractions (so  $5/4$  or  $\frac{5}{4}$ , not 1.25).*

(b; 5pts) Sketch the path in the  $xy$ -plane that represents the approximation carried out in part (a) and indicate its (path's) primary relation to the graph of the actual solution  $y = y(x)$  of the initial-value problem above.

(c; bonus 5pts) Use Euler's method with  $n = 3$  steps to estimate the value of  $y(-1)$ . *Show your steps clearly and use simple fractions.*

(d; bonus 10pts) Find the exact value of  $y(-1)$ . *Show your work and/or explain your reasoning.*

**Warning.** The last two questions are quite hard and very hard, respectively, and are subject to very harsh grading. Your time is likely to be better spent double- and triple-checking your work on the rest of the exam. If you do not see better ways of using your time on the exam, please answer these questions either on the facing page or on the back of this page and state below the questions where to find the answers.

**Problem 6 (25pts)**

(a; 5pts) What are the constant solutions of the differential equation

$$y' - (y^2 - 1)x = 0, \quad y = y(x) \quad ?$$

*Show your work and/or explain your reasoning.*

(b; 12pts) Find the general solution to the differential equation in (a). *Show your work and/or explain your reasoning.*

(c; 8pts) Sketch at least five solution curves, on the same plot of the  $xy$ -plane, representing every possible type of behavior of the solutions  $y = y(x)$  to the differential equation in (a). Justify the features exhibited on your plot. You can add comments to your plot to more clearly identify the features it is meant to exhibit. Hint: This question can be done with or without answering (b).