THERE ARE FIVE (5) PROBLEMS. THEY HAVE THE INDICATED VALUE.

SHOW YOUR WORK

DO NOT TEAR-OFF ANY PAGE

NO CALCULATORS       NO CELLS ETC.

ON YOUR DESK: ONLY test, pen, pencil, eraser.

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1. (50pts) Let $z_1 = 1 - i$, $z_2 = 3 - i$.

(a): Calculate $\bar{z}_1 \cdot z_2$ and $z_1/z_2$.

(b): Calculate $z_1^{1/3}$ and sketch the roots on a regular polygon.
2. **(50pts)** Calculate the limit if it exists:

(a) \[
\lim_{z \to i} \frac{z - i}{z(z^2 + 1)}.
\]

(b) \[
\lim_{z \to 0} \frac{\bar{z}^4}{z^3}.
\]
3. (50pts)

(1) Sketch the region given by:

\[ 0 \leq \text{Arg} z < \frac{3\pi}{4}, \quad 1 < |z| \leq 2. \]

(2) Find the image of the above region under the mapping \( w = z^2 \).
4. (50pts)

(a) Explain why the following function is analytic in its domain and calculate \( f'(z) \):

\[
f(z) = \frac{(iz - 1)^4}{(iz + 1)^4}.
\]

(b) If \( g(z) \) is an analytic function and \( f(z) = g(z) + \overline{g(z)} \) is also an analytic function, what can you say about \( g(z) \)? Explain your reason.
5. (50pts)

Find the points where the function is differentiable and then calculate the first order
derivative of the function at those points. Is the function analytic at those points?

(a)

\[ f(z) = (x^2 + (y + i)^2) + 2 \, i \, x(y + i) \]

(b)

\[ f(re^{i\theta}) = (\log r)^2 - \theta^2 + 2 \, i \, \theta \log r, \quad r > 0, 0 < \theta < 2\pi. \]