THERE ARE NINE (9) PROBLEMS. THEY HAVE THE INDICATED VALUE.

SHOW YOUR WORK

DO NOT TEAR-OFF ANY PAGE

NO CALCULATORS NO CELLS ETC.

ON YOUR DESK: ONLY test, pen, pencil, eraser.

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1. (50pts) (a) Find complex numbers $z$ such that $e^{1/z} = 2(1 - i)$.
   (b) Solve for $z$ such that $\cos(z) = 2$. 
2. (50pts)
(a): What’s the image $D_2$ of the region $D_1 = \{ z \in \mathbb{C}; 0 < \text{Re}(z) < \pi \}$ under the map $w = iz$?
(b): What’s the image of the region $D_2$ (from above) under the map $w = e^z$?
3. (50pts)

(a): Suppose $f(z) = u + iv$ is analytic. If we know that $u(z) = x^3 - 3xy^2$, what equations does $v$ satisfy? Solve them to get $v = v(z)$.

(b): Assume that $f$ is an entire function. If there is an analytic function $g(z)$ satisfying $f(z) = e^{g(z)}$, show that $f(z)$ has no zero point. Calculate $g'(z)$ in terms of the function $f(z)$. Reversely if $f(z)$ has no zero point on $\mathbb{C}$, does there exist such a $g(z)$?
4. (50pts) Calculate the following contour integrals.

(a): \[ \int_{|z|=3} \frac{\cos(z)}{z^5} \, dz. \]

(b): \[ \int_{0}^{2\pi i} \frac{1}{\cos^2(z)} \, dz \]
along any path from 0 to \(2\pi i\).

(c): \[ \int_{|z|=3} \bar{z} \, dz. \]
5. (50pts) (a): Find the Taylor series of the following function centered at 0.

\[ \frac{z}{(z - 2)^2}. \]

What’s the radius of convergence?

(b): Find the Taylor series of the above function centered at 1. What’s the convergence of radius?
6. (50pts) Find the Laurent series centered at 0 of the following function in the given region.

\[
\frac{2}{(z - 1)^2(z - 2)}
\]

(a) $|z| < 1$  (b) $1 < |z| < 2$  (c) $|z| > 2$
7. (50pts) Calculate the contour integrals using residues:
(a): \[ \int_{|z|=3} \frac{2}{(z - 1)^2(z - 2)} \, dz. \]
(b): \[ \int_{|z|=10} \frac{z^9}{z^5 + 1} \, dz. \]
8. (50pts) Classify the isolated singularities and calculate their residues:
   (a) \( \frac{\log z}{z - 1} \) at \( z = 1 \).
   (b) \( \cos(1/z) \) at \( z = 0 \).
   (c) \( \frac{\sin(z)}{(z - \pi)^4} \) at \( z = \pi \).
9. **(50pts)** Calculate the following integrals (a):
\[\int_{-\infty}^{\infty} \frac{x^2}{x^4 + 1} \, dx.\]
(b):
\[\int_{0}^{\infty} \frac{\cos(2x)}{(x^2 + 1)(x^2 + 4)} \, dx.\]
(c):
\[\int_{0}^{2\pi} \frac{d\theta}{3 + 2 \cos \theta}.\]
Scratch paper