Homework 7

1. Are the three functions \(\cos(2x)\), \(2\cos^2(x)\), \(5\sin^2(x)\) linearly independent or not?

2. Consider three functions \(y_1(x) = x^2 + x + 1\), \(y_2(x) = x + 1\), \(y_3(x) = x - 1\). Are they linearly dependent or not?

3-8. Find the general solutions in problems 3 through 8.
   3. \(y'' - 6y'' + 10y' = 0\).
   4. \(y^{(4)} - y = 0\).
   5. \(y^{(4)} + 2y'' + y = 0\).
   6. \(y^{(4)} - 2y'' + y = 0\).
   7. \(y^{(3)} + 2y'' + 2y' + y = 0\).
   8. \(y^{(3)} - 2y' - 4y = 0\).

9-11. In problems 9 through 11, a mass is attached to both a spring (with given spring constant \(k\)) and a dashpot (with given damping constant \(c\)). The mass is set in motion with initial position \(x_0\) and initial velocity \(v_0\).

   (a) Find the position function \(x(t)\) and determine whether the motion is overdamped, critically damped, or underdamped.

   (b) If it is underdamped, write the position function in the form \(x(t) = C_1 e^{-\alpha t} \cos(\omega t - \alpha t_0)\).

   (c) Find the undamped position function \(u(t) = C_0 \cos(\omega_0 t - \alpha_0)\) that would result if the mass on the spring were set in motion with the same initial position and velocity, but with the dashpot disconnected (so \(c=0\)).

   (d) Use Mathematica to plot the graphs that illustrate the effect of damping by comparing the graphs of \(x(t)\) and \(u(t)\).

9. \(m = 1, c = 4, k = 3; x_0 = 2, v_0 = -2\).
10. \(m = 1, c = 4, k = 4; x_0 = 2, v_0 = -2\).
11. \(m = 1, c = 4, k = 5; x_0 = 2, v_0 = -2\).

12. A body weighing 100 is oscillating attached to a spring and a dashpot. Its first two maximum displacements of 6 and 2 are observed to occur at times 1 and 2, respectively. Compute the damping constant and spring constant.