Homework 13

1. Find a power series solution of the given differential equation. Determine the radius of convergence of the resulting series, and identify the series solution in terms of familiar elementary functions.
   
   (a) \((x - 3)y' + y = 0.\)
   (b) \((1 - x^2)y' + 2xy = 0.\)
   (c) \(y'' - 4y = 0.\)
   (d) \(y'' + y = x.\)

2. For equations (a)-(e),

   (i) Find general solutions in powers of \(x\) of the differential equations. State the recurrence relation and the guaranteed radius of convergence in each case.

   (ii) Use power series to solve the initial value problem \(y(0) = 0, y'(0) = 1.\)

   (a) \((1 - x)y'' + y = 0.\)
   (b) \((x^2 - 1)y'' + 6xy' + 12y = 0.\)
   (c) \(y'' - 2xy' + 6y = 0.\) (Hermite equation)
   (d) \(y'' - 2xy' + 8y = 0.\) (Hermite equation)
   (e) \(y'' = xy.\) (Airy equation)