**MAT 203**     **FALL 2013**     **Practice MIDTERM I**

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**Total**     **260pts**
1. (40pts)

(a): Find the area of parallelogram on the plane with the following vertices
A(0, 0), B(7, 3), C(9, 8), D(2, 5).

(b): Calculate the cosines of the angles of the parallelogram.
2. (50pts) (a): Classify each of the following surfaces. Sketch the surface if possible.

(1) \( x^2 + y^2 - 2x + 4y = 0. \)
(2) \( y^2 + z^2 = 4. \)
(3) \( x^2 - z^2 - y^2 = 1. \)

(b): A quadric surface is a revolution surface obtained by rotating the curve \( x = -y^2 \) around the \( x \)-axis. Write down the equation for this surface and classify it.
3. (45pts)

Consider a point and a plane given by
\[ P = (1, 0, -1); \quad H : -4x + y + z = 4. \]

(a): Find the equation of the line passing through \( P \) and perpendicular to the plane \( H \).

(b): Find the intersection point of \( L \) with \( H \).

(c): Find the distance between the point and the plane.
4. (40pts)

Assume we have a vector valued function satisfying
\[ \vec{r}''(t) = -32\vec{j}, \quad r(0) = 8\vec{j} + 8\vec{k}, \quad \vec{r}'(0) = 8\vec{i} + 8\vec{j}. \]

(a): Find the expression for \( \vec{r}(t) \).

(b): Assume the curve \( C \) is described by the vector-valued function \( \vec{r}(t) \). Find the intersection points of \( C \) with the \( xz \)-plane.
5. (45pts)

Consider a motion is described by the smooth plane curve
\[ \vec{r}(t) = (2 \cos t)\hat{i} + (\sin t)\hat{j}. \]

(a): Sketch this curve. Calculate \(\vec{v}(t)\) and \(\vec{a}(t)\) for any \(t\).

(b): Find the unit tangent vector for any \(t\). Calculate the component of acceleration in the direction of \(\vec{T}\): \(a_T = \vec{a} \cdot \vec{T}\).

(c): Find the principal normal vector \(\vec{N}(2\pi/3)\) when \(t = \frac{2\pi}{3}\).
6. (40pts)

Calculate the length of the curve within the given interval
\[ \vec{r}(t) = (\cos^3 t) \hat{j} + (\sin^3 t) \hat{k}, \quad 0 \leq t \leq \pi/2. \]