MAT 203  FALL 2013  Practice Final

NAME:  ID:  RECITATION NUMBER:

THERE ARE TEN (10) PROBLEMS. THEY HAVE THE INDICATED VALUE.
SHOW YOUR WORK
DO NOT TEAR-OFF ANY PAGE
NO CALCULATORS   NO CELLS ETC.
ON YOUR DESK: ONLY test, pen, pencil, eraser.

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1
1. (50pts) Let \( f(x, y, z) = 9x^2 + y^2 - z^2 \).

(a): Sketch and describe the level surfaces \( f(x, y, z) = 9 \), \( f(x, y, z) = 0 \) and \( f(x, y, z) = -4 \).

(b): Find the tangent planes to the level surface \( f(x, y, z) = 0 \) at the points \((0, 5, 5)\) and \((0, 1, 1)\).

(c): Are the two planes in part (2) distinct? Explain how this relates to the geometry of the level surface.
2. (50pts) Consider the function $f(x, y) = \sin(x^2 + y^2)$.
(a): Compute $\nabla f$.

(b): A particle is moving on the path $(-t, t^2)$. Sketch the path for $0 \leq t \leq 2$. At $t = 1$, is $f$ increasing or decreasing along the path?

(c): Explain in words and draw the level curve $f = 0$. 
3. (50pts) Consider the curve $C$ defined by the equation:

$$(x - 1)^2 + 4y^2 = 4.$$ 

Find the point(s) on the curve $C$ which are closest to/furthest from the origin.
4. (50pts) Find the center of mass of snail-shaped region bounded by the curve $r = \theta^{1/3}, \ 0 \leq \theta \leq 2\pi$. Assume the mass density $\rho \equiv 1$. 
5. (50pts) Consider the solid $Q$ defined by the inequalities

$x^2 + y^2 = 1, z \leq -y, z \geq 0$.

(a): Write the triple integral $\iiint_Q dV$ into the iterated integral using the order $dzdydx$ and $dydxz$.

(b): Calculate the volume of $Q$. 

6. (50pts)

(a): Sketch the 3-dimensional region over which the integral
\[ \int_0^5 \int_0^{\sqrt{25-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{50-x^2-y^2}} z \, dz \, dy \, dx. \]

(b): Convert this integral into cylindrical and spherical coordinates.

(c): Compute the integral.
7. (50pts)

(a): Is the following fluid vector field compressible or not?
\[ \vec{F} = e^x \sin(y + z) \hat{i} + e^x \cos(y + z) \hat{j} + (x^2 + y^2) \hat{k}. \]

(b): Is the following vector field conservative or not?
\[ \vec{F} = (y + z) \hat{i} + z \hat{j} + (x + y) \hat{k}. \]
8. (50pts)

Consider the circle $C : x^2 + y^2 = a^2$. (a): Find the area of lateral surface with height

function $z = |x| + |y|$ over the curve $C$. 
(b): Consider the vector field:

\[ \vec{F} = \frac{-y\vec{i} + x\vec{j}}{x^2 + y^2}, (x, y) \neq (0, 0). \]

Sketch the vector field, and calculate the circulation of \( \vec{F} \) around the curve \( C : x^2 + y^2 = a^2 \) in the \textbf{counter-clockwise} direction. What’s the circulation in the \textbf{clockwise} direction?
9. (50pts)

Let $\vec{F}$ be the vector field

$$\vec{F} = (2x + y - 1, x + 2y).$$

(a): Is $\vec{F}$ conservative or not?

(b): If yes, find a potential function $f(x, y)$. 
(c): A particle is moving from the point \((0, 0)\) to a point \((p, q)\) in a straight line. For which values of \(p, q\) is the work
\[\int_{(0,0)}^{(p,q)} \vec{F} \cdot d\vec{r}.\]
minimal?
10. (50pts)

Use Green’s theorem to calculate the work done by $\vec{F}$

$$\vec{F} = (4x^2 - 2y^2)i + (2x^2 - 4y^2)j.$$

in moving a particle once counterclockwise around the boundary of a triangle with vertex: $(0, 0)$, $(1, 0)$, $(0, 1)$?