Homework Problems Mat 331
Set no. 3, November 26,2003
Due December 8, 2003

(1) Find all common solutions $(\mod 12)$ (or show that there are none) to

$$4x + y \equiv 6(\mod 12), \quad x + 4y \equiv 9(\mod 12)$$

(2) Find all positive integers less than 1000 which leave the remainder 1 when divided by 2, 3, 5 and 7.

(3) Find a reduced system $(\mod 20)$ and give $\phi(20)$

(4) Show that $3^3 \equiv -4(\mod 31)$ and use this to show that $3^{10} \equiv -6(\mod 31)$. Use this result and Euler's theorem to show that

$$3^{341} \not\equiv 3(\mod 31)$$

and therefore

$$3^{341} \not\equiv 3(\mod 341).$$

(5) Show that if p is a prime, and a is an integer, and k is a non-negative integer, then

$$a^{1+k(p-1)} \equiv a(\mod p).$$

(6) Find $\phi(n)$ for $n = 20, 60, 63, 341$ and 561.

(7) Let x be the smallest positive integer such that $2^x \equiv 1(\mod 63)$. Find x and verify that $x|\phi(63)$.