Homework Problems Mat 331
Set no. 1 September 10, 2003
Due September 19, 2003

(1) Use *Maple* to write $x^5-2x^4-10x^3+20x^2-16x+32$ as a product of *exact* linear factors. By exact, I mean you should leave any non-rational factors expressed as radicals; do not approximate terms like $\sqrt{3}$ as 1.73205, etc.

(2) Draw a graph showing both $\cos(x)$ and its fifth Taylor polynomial (that is, $1-\frac{1}{2!}x^2+\frac{1}{4!}x^4$) for x between -4 and 4. What degree of Taylor polynomial seems to be needed to get good agreement in this range? **Hint:** Use a variation of the command *convert(taylor(cos(x),x,5),polynom)* to make this work. Think of a suitable way to demonstrate that the approximation you have taken is "good"- what is a good definition of "good" here?.

(3) Consider the planar curve γ defined by $x^2y^3+y^2+y-2e^x=0$ Using **only** *Maple*, find the slope of the tangent line to the curve at $(0,1)$. Then plot the curve and the tangent line on the same graph.

Hint: You might want to use *implicitplot* from the library *plots*. You might find *implicitdiff* helpful, too.

(4) Plot the function $f(x)=2\sin x-x^3-\frac{1}{5}$, for $x\in[-4,4]$. Find all the zeros of the function with an accuracy of 20 decimal digits.

Hint: See *Digits, fsolve*.

(5) Define a *Maple* function g that, given a positive integer k yields the sum of the first k primes. What is k such that $g(k)\leq 100,000$ but $g(k+1)>100,000$?.

Hint: You might find *sum* and *ithprime* helpful.

(6) Use the Taylor expansion of $\arctan x$ near the point $x=\frac{1}{\sqrt{3}}$ to compute the value of π to 30 places. How many terms are needed to compute the value to 50 places?