MAT 402

Homework IV
Due April 9th. 2019. Show all your work

(1) Let $f; \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = \sin(z) + z$. Show that, for each $n \in \mathbb{Z}$, $(2n+1)\pi$ is a super-attracting fixed point of f, and hence belongs to the Fatou set $F(f)$, and $2n\pi$ is a repelling fixed point of f, and hence belongs to the Julia set $J(f)$.

(2) Let $f : \mathbb{C} \to \mathbb{C}$ be a non-affine, non-constant entire function. Show that the sets $I(f)$ and $K(f)$ are both completely invariant, i.e., $f\big(I(f)\big) = I(f)$ and $f\big(K(f)\big) = K(f)$. More precisely, you should prove that each set is contained in the other. (The set $I(f)$ is defined in section 2.6, page 26.)

(3) Let $f : \mathbb{C} \to \mathbb{C}$ be a non-affine , non-constant entire function, and suppose that the family of iterates $\{f^n\}_{n=1}^{\infty}$ is equicontinuous at z. Show that the family is also equicontinuous at $f(z)$.