MAT 402

Homework III

Due March 14th. 2019. Show all your work

(1) Suppose that f is the rational map $f(z) = \dfrac{z^2+2}{z^2+2z-3}$. Use the technique of Lemma 3.2.5 to find the spherical derivative of f at the pole $z = 1$.

(2) Show that $M(z) = (az+b)/(cz+d)$ with $ad - bc = 1$ is an isometry of the Riemann sphere if and only if it is of the form
$$M(z) = \frac{az+b}{\overline{a} - \bar{b}z} \ ,$$
where $a,\, b \in \mathbb{C}$ with $|a|^2 + |b|^2 \neq 0$.

(3) Let f be the inversion map $f(z) = 1/z$. Prove that f maps circles through 0 to straight lines, straight lines to circles through 0, and other circles again to circles.
Hint: Write the equation of a circle as $|z-a|^2 = (z-a)(\overline{z} - \overline{a}) = r^2$.