

MAT 402

Homework II

Due March 5th, 2019. Show all your work

- (1) Find a conjugacy between the tent map $T : [0, 1] \rightarrow [0, 1]$

$$T(x) = \begin{cases} 2x & x \leq 1/2 \\ 2 - 2x & x > 1/2 \end{cases}$$

and $G(x) = 2x^2 - 1$ on the interval $[-1, 1]$.

Hint: You may want to think of “angle doubling”.

- (2) Consider the entire function $f : \mathbb{C} \rightarrow \mathbb{C}$, $f(z) = \frac{1}{2}(e^z - 1)$.
- (a) Show that 0 is an attracting fixed point of f . Show that f has exactly one other fixed point p on the real axis, and that $p > 0$. (*Hint:* the function f is convex.)
 - (b) Show that $x < f(x) < 0$ for $x < 0$, that $0 < f(x) < x < p$ whenever $0 < x < p$, and that $f(x) > x > p$ whenever $x > p$.
 - (c) Deduce that $(-\infty, p) \subseteq \mathcal{B}(0)$ and that $(p, \infty) \subseteq \mathcal{I}(f)$.
 - (d) Find an absorbing domain for the attracting fixed point at 0.
- (3) Prove that if K is a compact set such that f_n are continuous functions on K , and if f_n converges uniformly to some function f on K , then f_n are equicontinuous on K .