**MAT 402**

**Homework I**
**Due February 7th. 2019**

(1) Study the dynamics of the map $f_\lambda(x) = \lambda x(1-x)$ for $\lambda = 2$. By "studying the dynamics of a map" I mean that you need to find all attracting and repelling fixed points of the map $f_\lambda$. Then for each attracting point determine all the points in $[0,1]$ which converge to it. Justify your answer fully.

(2) Let $Q_c(x) = x^2 + c$, where $x \in \mathbb{R}$. Prove that if $c < 1/4$ there is a unique $\lambda > 1$ such that $Q_c$ is topologically conjugate to $f_\lambda(x) = \lambda x(1-x)$ via a map of the form $h(x) = \alpha x + \beta$.

(3) In the Feigenbaum bifurcation diagram for $f_\lambda$, the Feigenbaum point (which is the limit of the period doubling cascade) has coordinate $\lambda \approx 3.57$.

In the case of the Mandelbrot set, there is also a limit of a period doubling cascade (also called the Feigenbaum point) with coordinate $c \approx -1.401155\ldots$ What is the change of coordinate that respects the dynamics and takes $f_\lambda$ to $Q_c$?