The exam will cover chapters 1 and 2, and section 3.1 as well as linear transformations as we've discussed them in lecture. The following is a list of things I definitely want you to know. It is certainly not exhaustive (I'm not about to retype 100 pages of textbook!), but hopefully gives you some idea. Note that some questions here are to demonstrate an idea, and so are chosen to be simplistic. Others are at what I feel is an appropriate level for the exam. You would do well to review your homeworks, especially the problems that you struggled with.

- Systems of equations and Gauss elimination. You should be able to completely reduce a system of equations. You will also benefit from understanding what you are doing and why in this process.
- You should be comfortable with matrix algebra—adding and rescaling vectors, matrix multiplication, linearity of these operations.
- The definition of a linear transformation is the one thing that I have asked you to memorize in this course. It's that important. That doesn't mean that there will be a question like "state the definition of linearity" on the exam. You obviously need to understand the definition, well enough that you can apply it.

  For example, consider a linear map $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ for which

$$S[e_1] = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \qquad S[e_2] = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix}.$$

  Since S is linear, you can determine it's value for any vector. What is

$$S\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]?$$

- Linear maps always have a matrix representation: for fixed linear $T\colon \mathbb{R}^n \to \mathbb{R}^m$, there is a corresponding matrix A so that $T[v] = Av$ for all vectors v in $\mathbb{R}^n$. However, linear maps don't always come with the matrix given. A good example from the homework is the linear map $R\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by reflection across a line (2.2.16(b)). That is a perfectly valid and useful description of a linear map, but sometimes it is also useful to know the corresponding matrix. What 2x2 matrix corresponds to R? (See next bullet point for a hint.)
- Homework 3 was a difficult one. I hope that at this point you can look back and find it a little easier, now that we've had some more time to think about linearity. I would reccomend reviewing it specifically, as there was a lot of meat there.

  One particularly useful fact that was helpful for that homework: for fixed linear $T\colon \mathbb{R}^n \to \mathbb{R}^m$, the corresponding matrix A is the matrix whose n columns are given by $T[e_1], \ldots, T[e_n]$. (Recall the standard basis vectors $e_1, \ldots, e_n$ of $\mathbb{R}^n$.) This is one of those facts that sounds complicated, but is not, and knowing it will make your life much easier. To check your understanding, make sure you understand why the linear map S above is represented by the matrix

$$\begin{pmatrix} 2 & -1 \\ -3 & 2 \\ 6 & -4 \end{pmatrix}.$$

- Another useful theorem: a linear map $T\colon \mathbb{R}^n \to \mathbb{R}^m$ is uniquely determined by it's value on the n vectors $e_1, \ldots, e_n$. Why is this true? (Hint: T is a linear map.) Use the theorem and planar geometry to show that the two following linear maps are equal:

1) $R_1 \colon \mathbb{R}^2 \to \mathbb{R}^2$ given by the rule "take v and rotate it by $2\pi/3$ degrees counterclockwise."

2) $R_2 \colon \mathbb{R}^2 \to \mathbb{R}^2$ given by the rule "take v, reflect across the x-axis, then reflect the result across L", where L is the line through the origin at angle $\pi/3$ from the x-axis

- A linear function $T \colon \mathbb{R}^n \to \mathbb{R}^m$ is, like any function, a rule that assigns a vector in $\mathbb{R}^m$ to each vector in $\mathbb{R}^n$. Can you express R_2 from the previous bullet point as the composition of 2 linear maps?
- On the other hand, the composition of linear maps works nicely together with matrix multiplication: If T is represented by A and S represented by B, then TS is represented by AB.
- Invertibility. A linear map is invertible if and only if the representing matrix is invertible. We discussed in lecture how it is easier to show that a linear map *is not* invertible and that a matrix *is* invertible. For example, the linear map $\mathbb{R}^3 \to \mathbb{R}^3$ given by the rule "project v to the xy-plane" is not invertible. Why? On the other hand, the matrix
$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
is invertible. Why? What is its inverse? (Hint: the inverse is upper triangular.)
- The formula for inverses of 2x2 matrices is useful.
- Recall the method for using Gauss-Jordan elimination to find the inverse of a matrix.
- There will likely be a question involving the image or the kernel of a linear map. I may remind you of the definition, but you will need to understand it well enough to apply it. For example, it would be reasonable for me to ask you to demonstrate that $\ker(T)$ for some given linear map T is a subspace. (You have to check that it is closed under addition and rescaling. We did the proof in class. Don't memorize the proof.)
- Throughout the lecture, there were several theorems, often given with a proof. You should know/understand and be able to apply the theorems. Their purpose is to save you work, because applying a theorem is better than reproving it in a specific instance. You will not be asked to recreate the proofs, but you should know them well enough that you could follow the logic if provided. If I gave you a proof in lecture, it was because there was an idea in it that I think is useful for you to know. Those ideas may well appear on the exam. For example, many of the theorems we've seen shared very similar proofs—essentially, use the two properties of linearity and the decomposition of a vector into the standard basis—and you would do well to understand that general idea. Understanding the proofs is a good way to gain that understanding.