- This review is necessarily not exhaustive. I recommend looking back over the homework, especially the problems that were graded and the ones that you found difficult. I also offered exercises in lecture, which should give you an idea of what I think is interesting to ask, and what I think is reasonable to ask.
- Consider a linear map $T\colon \mathbb{R}^5 \to \mathbb{R}^3$ and vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in \mathbb{R}^5$. Suppose that $T[\vec{v}_1], T[\vec{v}_2], T[\vec{v}_3]$ are linearly independent. Are $\vec{v}_1, \vec{v}_2, \vec{v}_3$ independent? Hint: use the definition of linear independence.
- Make sure you know and understand the statement of the rank nullity theorem. It's consistently useful, because it often quickly tells you the dimension of either the kernel or the image of a map.
- Knowing the dimension of a space makes finding a basis half as difficult: if $\dim(V) = n$ and vectors $\vec{v}_1, \dots, \vec{v}_n$ either 1) span V or 2) are independent, then both 1 and 2 are true, meaning they form a basis for V.
- Consider the linear map $T\colon P_3 \to P_3$ given by the rule $T[f(t)] := f(t) + tf'(t) + f''(t)$. Use the standard basis of P_3 to express T as a matrix. Draw the coordinate square that gives rise to this matrix.
- Find a basis for the kernel of T from the previous bullet. Find a basis for the image.
- Let P_2 be the vector space of quadratic polynomials. Let $\mathcal{B} = \{1+t, t+t^2, 1+t^2\}$ be a basis for P_2. What are the coordinates of $2+t+2t^2$ in this basis? What are the coordinates of the general quadratic $a_0 + a_1 t + a_2 t^2$?
- Express the matrix

$$A = \begin{pmatrix} 3 & 0 & -2 \\ -7 & 0 & 4 \\ 4 & 0 & -3 \end{pmatrix}$$

in terms of the basis

$$\mathcal{B} = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \right\}.$$

Draw the coordinate square.
- Write the change of basis matrix from the basis $\mathcal{B}$ in the previous bullet to the basis

$$\mathcal{A} = \left\{ \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \right\}.$$

How about the change of basis matrix from $\mathcal{A}$ to $\mathcal{B}$.
- Make sure you understand the criterions for when a linear map T is an isomorphism. There is a handy table on page 183 of the book. Don't memorize it, but do make sure that you understand the pieces.
- Consider the linear map given by reflection across the plane $x_1 + x_2 + x_3 = 0$ in $\mathbb{R}^3$. Reason geometrically to find a basis of eigenvectors for this map. (Note that the vector $(1\ 1\ 1)^T$ is perpendicular to this plane.)
- Consider a linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^n$ which has a basis consisting of eigenvectors $\{\vec{v}_1, \dots, \vec{v}_n\}$ with corresponding eigenvalues $\lambda_1, \dots, \lambda_n$. What is the matrix of T in this basis? Why?

- What is the determinant of
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 1 & 3 & 3 & 3 \\ 1 & 1 & 1 & 4 & 4 \\ 1 & 1 & 1 & 1 & 5 \end{pmatrix}$$
- Diagonalize $A = \begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix}$. First you find the eigenvalues, then the eigenvectors, then you use that to form the matrix S whose columns are the eigenvectors. Then $D = S^{-1}AS$ will be diagonal.
- What is A^{10}? Here A is the matrix from the previous problem. Don't multiply it out. Instead, note that D^{10} is easy to express, and check that $A^{10} = (SDS^{-1})^{10} = SD^{10}S^{-1}$.