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PUBLICATIONS

[1] C. J. Bishop and J. K. Wetterer Planktivore prey selection: the relative field volume model vs. the apparent size model. *Ecology*, 66(2):457, 1985.

We compare two models for how fish choose what to eat. We show the predictions of the two models agree in deep water (fish select from a sphere), but differ in shallow water (fish select from a truncated sphere), and report on an experiment to distinguish the cases.

[2] C. J. Bishop. A counterexample in conformal welding concerning Hausdorff dimension. *Michigan Math. J.*, 35(1):151–159, 1988.

I construct an example of a closed Jordan curve with the property that harmonic measure (i.e., first hitting distribution of Brownian motion) for the two sides are comparable for every subset, but the curve has Hausdorff dimension > 1. This showed that even strong conditions on harmonic measure could not be used to characterize rectifiable curves.

[3] C. J. Bishop. An element of the disk-algebra that is stationary on a set of positive length. *Algebra i Analiz*, 1(3):83–88, 1989.

This answered a question of Havin and Makarov, by constructing an example of a non-constant analytic function on the unit disk whose derivative extends to be zero on a positive length subset of the unit circle.

[4] C. J. Bishop. Constructing continuous functions holomorphic off a curve. *J. Funct. Anal.*, 82(1):113–137, 1989.

This gives an explicit construction of certain space filling curves that arise naturally in function theory (they were previously known to exist by a result of Browder and Wermer using an indirect Hahn-Banach argument, but no one had "seen" one before).

[5] C. J. Bishop. Approximating continuous functions by holomorphic and harmonic functions. *Trans. Amer. Math. Soc.*, 311(2):781–811, 1989.

Various results are proven about algebras generated by harmonic functions on plane domains which generalize well known results on the unit disk. For example, the closed algebra generated by the bounded holomorphic functions and the complex conjugate of a single non-constant holomorphic function contains every uniformly continuous function on the domain.

[6] C. J. Bishop, L. Carleson, J. B. Garnett, and P. W. Jones. Harmonic measures supported on curves. *Pacific J. Math.*, 138(2):233–236, 1989.

We geometrically characterize when the harmonic measure from two opposite sides of a closed curve are either mutually singular or mutually continuous. For example, if we start Brownian motions on opposite sides of fractal curve such as the von Koch snowflake, there

are two disjoint sets which absorb the paths, i.e., the two hitting measures are mutually singular. I am fond of this paper because it gives me a co-authorship with Lennart Carleson.

[7] C. J. Bishop. Bounded functions in the little Bloch space. *Pacific J. Math.*, 142(2):209–225, 1990.

Sarason gave a non-constructive proof that there exist Blaschke products in the little Bloch space (a class of holomorphic functions on the disk whose derivatives grow slowly near the boundary) but no explicit example was known. This paper gives such an example and characterizes all such examples in terms of their zero sets. This result has been extended to various spaces by others.

[8] C. J. Bishop. Conformal welding of rectifiable curves. *Math. Scand.*, 67(1):61–72, 1990.

This answers a question of Walter Hayman. It constructs an example of a rectifiable curve that has a certain pathological property with respect to harmonic measure.

[9] C. J. Bishop and P. W. Jones. Harmonic measure and arclength. *Ann. of Math. (2)*, 132(3):511–547, 1990.

This paper proves a conjecture of Øksendal that has various consequence in geometric function theory. We show that a set of zero length which lies on a rectifiable curve has zero harmonic measure in any simply connected domain.

[10] C. J. Bishop and T. Steger. Three rigidity criteria for PSL$(2, \mathbf{R})$. *Bull. Amer. Math. Soc. (N.S.)*, 24(1):117–123, 1991.

A summary of my Acta. Math. paper with Steger. See [14] below.

[11] C. J. Bishop. A characterization of Poissonian domains. *Ark. Mat.*, 29(1):1–24, 1991.

A domain is called Poissonian if every bounded harmonic function on the domain is the Poisson extension of a bounded function on the boundary. This paper provides a characterization of such domains in terms of the geometry of the boundary. It also proves some bounds on the complexity of the Martin boundary of any Euclidean domain. Conjectures made in this paper led to work of Tsirelson on the impossibility of stochastic processes with certain symmetries. Some of the result of this paper have been generalized by Azzam, Mourgoglou, and Tolsa.

[12] C. J. Bishop. Brownian motion in Denjoy domains. *Ann. Probab.*, 20(2):631–651, 1992.

This considers a Cauchy process on the real line (which can be obtained from a Brownian motion in the plane restricted to the times when it is on the line). We show that for sets E of positive length, such a process has a well defined "direction of approach" to its first hitting of E. This answers a question of Chris Burdzy.

[13] C. J. Bishop. Some questions concerning harmonic measure. In *Partial differential equations with minimal smoothness and applications (Chicago, IL, 1990)*, volume 42 of *IMA Vol. Math. Appl.*, pages 89–97. Springer, New York, 1992.

A collection of open problems. One has been solved by Sunhi Choi, another by Azzam, Hofmann, Martell, Mayboroda, Mourgoglou, Tolsa and Volberg.

[14] C. J. Bishop and T. Steger. Representation-theoretic rigidity in PSL$(2, \mathbf{R})$. *Acta Math.*, 170(1):121–149, 1993.

Mostow rigidity implies that any two lattice inclusions of an abstract group into a connected simple Lie group are equivalent, if that group is not $PSL(2, R)$ (a better known consequence is that the geometry of a 3-dimensional hyperbolic manifold is determined by the fundamental group). This famous result fails for $PSL(2, R)$ because a surface can carry many non-equivalent Riemann surface structures. However, we show that a version of Mostow's theorem is still true: two representations of $PSL(2, R)$ restricted to two lattices are equivalent iff they are the same representation and the two lattices are conjugate.

[15] C. J. Bishop. An indestructible Blaschke product in the little Bloch space. *Publ. Mat.*, 37(1):95–109, 1993.

Another explicit construction of a holomorphic function with odd properties.

[16] C. J. Bishop. How geodesics approach the boundary in a simply connected domain. *J. Anal. Math.*, 64:291–325, 1994.

This answers a question of Chris Burdzy about the geometric properties of a hyperbolic geodesics in a plane domain. In particular, if a geodesics passes within a small Euclidean distance of a boundary point, does it have to hit the boundary near this point? If not, how far away can the hitting point be on average? The most amusing aspect of this paper is that the answer involves the convergence or divergence of an integral of the form $\int_0^1 f(t)^{2/9} dt/t$.

[17] C. J. Bishop and P. W. Jones. Harmonic measure, L^2 estimates and the Schwarzian derivative. *J. Anal. Math.*, 62:77–113, 1994.

This simplifies and extends results from [8] and is part of a broader program to use "L^2 techniques" on sets rather than functions. Peter and I develop an analog of Littlewood-Paley theory based on the Schwarzian derivative (in place of the usual derivative) and use it to prove a number of conjectures from function theory, including a geometric characterization of "BMO domains" and an a.e. characterization of tangent points of a curve. Surprisingly, this paper is now cited in various papers dealing with large data sets because of a lemma that characterizes when a set in the plane can be approximated by a curve of finite length. This result was later generalized by others to estimate how well a set of points in a high dimensional space can be approximated by a lower dimensional (but not necessarily linear) submanifold.

[18] C. J. Bishop. Some homeomorphisms of the sphere conformal off a curve. *Ann. Acad. Sci. Fenn. Ser. A I Math.*, 19(2):323–338, 1994.

This gives some examples of some unusual homeomorphisms of the 2-sphere to itself that are conformal except on a small set (dimension 1).

[19] C. J. Bishop. A counterexample concerning smooth approximation. *Proc. Amer. Math. Soc.*, 124(10):3131–3134, 1996.

This answers a question of Stegenga and Smith about approximations in Sobolev spaces.

[20] C. J. Bishop. A distance formula for algebras on the disk. *Pacific J. Math.*, 174(1):1–27, 1996.

This gives a characterization of certain algebras of functions on the unit disk which are generated by harmonic and holomorphic functions.

[21] C. J. Bishop. Minkowski dimension and the Poincaré exponent. *Michigan Math. J.*, 43(2):231–246, 1996.

This gives a method of estimating the Hausdorff dimension of fractals known as Kleinian limit sets. The techniques here were adapted by Avila and Lyubich to study Julia sets.

[22] C. J. Bishop. On a theorem of Beardon and Maskit. *Ann. Acad. Sci. Fenn. Math.*, 21(2):383–388, 1996.

This gives a characterization of geometrically finite Kleinian group. It is a cleaner version of an earlier result by Beardon and Maskit.

[23] C. J. Bishop. Some characterizations of $C(\mathcal{M})$. *Proc. Amer. Math. Soc.*, 124(9):2695–2701, 1996.

This gives a simple characterization of which continuous functions on the unit disk are in the algebra generated by all bounded harmonic functions. This happens iff the level sets of the function can be approximated by curves on which arc length is a Carleson measure.

[24] C. J. Bishop. Geometric exponents and Kleinian groups. *Invent. Math.*, 127(1):33–50, 1997.

Maskit showed the diameters of the components of the ordinary set sum the fourth power and conjectured the second power suffices; this is proven here. This shows that most components are not "long and skinny" in a precise way.

[25] C. J. Bishop and Y. Peres. Packing dimension and Cartesian products. *Trans. Amer. Math. Soc.*, 348(11):4433–4445, 1996.

This gives optimal estimates for the dimension of the product of two sets. Previous results gave bounds which might or might not be sharp for a particular set, but this gives the sharp result for any set.

[26] C. J. Bishop and Peter W. Jones. Hausdorff dimension and Kleinian groups. *Acta Math.*, 179(1):1–39, 1997.

This is my most cited paper. This proves three main results: the critical exponent of a Kleinian group equals the Hausdorff dimension of its radial limit set, the limit set of a finitely generated Kleinian group has Hausdorff dimension two iff the group is geometrically infinite, and Hausdorff dimension of the limit set is upper semi-continuous. Each of these was a well known conjecture in the field. Our results have been extended to other settings such as symmetric spaces, variable curvature, Gromov hyperbolic groups and discrete quasiconformal groups.

[27] C. J. Bishop and P. W. Jones. The law of the iterated logarithm for Kleinian groups. In *Lipa's legacy (New York, 1995)*, volume 211 of *Contemp. Math.*, pages 17–50. Amer. Math. Soc., Providence, RI, 1997.

We prove a conjecture of Dennis Sullivan regarding the exact Hausdorff dimension of certain Kleinian group limit sets by exploiting the connections between harmonic functions and martingales. We also answer a question of Curt McMullen regarding absolute continuity of conjugations between certain types of groups.

[28] C. J. Bishop and P. W. Jones. Wiggly sets and limit sets. *Ark. Mat.*, 35(2):201–224, 1997.

We show that if a connected set deviates uniformly from a line segment at all points and all scales, then it must have dimension strictly larger than 1. The proof is a rather intricate argument involving the so called "traveling salesman theorem" of Peter Jones, but so far no one has yet found an easier proof of this simple sounding result.

[29] C. J. Bishop, P. W. Jones, Robin Pemantle, and Yuval Peres. The dimension of the Brownian frontier is greater than 1. *J. Funct. Anal.*, 143(2):309–336, 1997.

We prove that the frontier of a Brownian motion (i.e., the boundary of its unbounded complementary component) has Hausdorff dimension strictly bigger than one, providing the first evidence for a conjecture of Mandelbrot that the dimension equals 4/3. The full conjecture was later proven using Stochastic Loewner Evolutions (SLE) as developed by Lawler, Schramm and Wendelin (resulting in Fields medal for the latter). One fun aspect of our proof is a nested decomposition of the plane into fractal tiles, instead of the usual square grid (we needed to avoid straight lines in certain boundary estimates). I suspect this might be useful for other problems.

[30] C. J. Bishop. Quasiconformal mappings which increase dimension. *Ann. Acad. Sci. Fenn. Math.*, 24(2):397–407, 1999.

This answers a question of Heinonen by showing that the dimension of any set $E \subset R^n$ with $0 < \dim(E) < n$ can be increased by taking a quasiconformal image. This is interesting because the dimension cannot always be lowered (the infimum of dimension under quasiconformal maps is called the conformal dimension of the set and investigating it is currently a very active area).

[31] C. J. Bishop. A quasisymmetric surface with no rectifiable curves. *Proc. Amer. Math. Soc.*, 127(7):2035–2040, 1999.

Builds a pathological surface in 3-space that is nice in certain respects, but which contains no connected set of finite length. This answered a question of Rohde. This result was extended by David and Toro.

[32] C. J. Bishop, A. Böttcher, Yu. I. Karlovich, and I. Spitkovsky. Local spectra and index of singular integral operators with piecewise continuous coefficients on composed curves. *Math. Nachr.*, 206:5–83, 1999.

This discusses the properties of a certain class of operators. My contribution consisted of the proof of certain estimates regarding harmonic measure.

[33] C. J. Bishop and J. T. Tyson. Conformal dimension of the antenna set. *Proc. Amer. Math. Soc.*, 129(12):3631–3636, 2001.

We compute the minimal possible dimension over all possible quasiconformal images of a certain fractal. In trying to understand how the dimension of a set can be lowered by QC mappings, there are very few non-trivial cases where we actually know what the minimum is. This provides one such example. This has been extended to metric spaces by Azzam, who also answers a question from this paper.

[34] C. J. Bishop and J. T. Tyson. Locally minimal sets for conformal dimension. *Ann. Acad. Sci. Fenn. Math.*, 26(2):361–373, 2001.

This gives more examples relevant to dimension lowering by quasiconformal maps.

[35] C. J. Bishop. Bi-Lipschitz homogeneous curves in $\mathbb{R}^2$ are quasicircles. *Trans. Amer. Math. Soc.*, 353(7):2655–2663 (electronic), 2001.

This proves that if a Jordan curve is biLipschitz homogeneous then it is "nice" (i.e., satisfies Ahlfors' 3-point condition). Homogeneous means that any point in the set can be mapped to any other point by a map which is biLipschitz on the set. This had been an open problem in the field. It is open whether an exotic continua can be biLipschitz homogeneous.

[36] C. J. Bishop. Divergence groups have the Bowen property. *Ann. of Math. (2)*, 154(1):205–217, 2001.

This completes a series of papers by Rufus Bowen, Dennis Sullivan, Kari Astala and Michel Zinsmeister by showing that a Fuchsian group is divergence type iff it has Bowen's property (this says that any deformation of the group has a limit set which is either a circle or has Hausdorff dimension > 1). A novelty of the proof is the use of Dennis Sullivan's "Convex Hull Theorem" in hyperbolic 3-space and I give a new proof of Sullivan's result.

[37] C. J. Bishop. BiLipschitz approximations of quasiconformal maps. *Ann. Acad. Sci. Fenn. Math.*, 27(1):97–108, 2002.

This that shows quasiconformal maps in two dimensions can be approximated by bi-Lipschitz maps in a certain precise sense.

[38] C. J. Bishop. Quasiconformal mappings of Y-pieces. *Rev. Mat. Iberoamericana*, 18(3):627–652, 2002.

This describes the optimal way to deform a Riemann surface by shrinking closed loops. It is used in [39]. It has also been quoted widely in the literature on Teichmuller spaces of infinite area Riemann surfaces.

[39] C. J. Bishop. Non-rectifiable limit sets of dimension one. *Rev. Mat. Iberoamericana*, 18(3):653–684, 2002.

Answers a question of Astala and Zinsmeister by constructing certain deformations of Fuchsian groups whose limit sets are non-rectifiable curves of dimension 1.

[40] C. J. Bishop and P. W. Jones. Compact deformations of Fuchsian groups. *J. Anal. Math.*, 87:5–36, 2002. Dedicated to the memory of Thomas H. Wolff.

We show that a deformation of a Fuchsian group which only changes the conformal structure on a compact set gives a limit set where the escaping limit set had dimension 1.

[41] C. J. Bishop. Quasiconformal Lipschitz maps, Sullivan's convex hull theorem and Brennan's conjecture. *Ark. Mat.*, 40(1):1–26, 2002.

This gives a new proof of a result of Dennis Sullivan about convex sets in hyperbolic space and gives applications to conformal maps in the plane. One such is the fact that any simply connected plane domain can be mapped to a disk by a Lipschitz homeomorphism.

[42] C. J. Bishop, V. Ya. Gutlyanskiĭ, O. Martio, and M. Vuorinen. On conformal dilatation in space. *Int. J. Math. Math. Sci.*, (22):1397–1420, 2003.

This is my second most cited paper. This shows that if the dilatation of a quasiconformal map in space satisfies certain integrability conditions then the map is pointwise differentiable.

[43] C. J. Bishop. Big deformations near infinity. *Illinois J. Math.*, 47(4):977–996, 2003.

This proves the existence of certain deformations of a Fuchsian group. These are used in [44] to study the analytic dependence of a limit set as a function of a deformation parameter.

[44] C. J. Bishop. δ-stable Fuchsian groups. *Ann. Acad. Sci. Fenn. Math.*, 28(1):153–167, 2003.

This introduces the idea of a δ-stable group and gives examples of such things.

[45] C. J. Bishop. An explicit constant for Sullivan's convex hull theorem. In *In the tradition of Ahlfors and Bers, III*, volume 355 of *Contemp. Math.*, pages 41–69. Amer. Math. Soc., Providence, RI, 2004.

Improves the constant ($K = 7.82$) in Dennis Sullivan's convex hull theorem. The size of this constant has implications for planar conformal mappings. The constant has since been improved by Bridgeman, Canary and Yarmola.

[46] C. J. Bishop. The linear escape limit set. *Proc. Amer. Math. Soc.*, 132(5):1385–1388 (electronic), 2004.

Proves some results concerning the Hausdorff dimension of special limit sets. In particular, the dimension of the limit set either equals the dimension of the bounded geodesics, or the dimension of the geodesics that tend to infinity as fast as possible (linear speed).

[47] C. J. Bishop. Orthogonal functions in H^∞. *Pacific J. Math.*, 220(1):1–31, 2005.

This answers a well known question of Walter Rudin: if f is holomorphic on the unit disk and the sequence of powers $f, f^2, f^3, \dots$ are orthogonal, must f be an inner function (i.e., $|f| = 1$ a.e. on the circle)? In fact, I provide large families of counter-examples, essentially one for every finite, radial measure in any annulus. An independent solution was obtained by Carl Sundberg.

[48] C. J. Bishop. Boundary interpolation sets for conformal maps. *Bull. London Math. Soc.*, 38(4):607–616, 2006.

This characterizes interpolating sets for conformal maps of the disk, i.e., sets E on the circle so that given any homeomorphism there is a conformal map on the disk whose restriction to E agrees with the given map.

[49] C. J. Bishop. A criterion for the failure of Ruelle's property. *Ergodic Theory Dynam. Systems*, 26(6):1733–1748, 2006.

We give examples to show that even for divergence type groups, the dimension of the limit set need not be an analytic function of the deformation.

[50] C.J. Bishop. Harmonic measure by Garnett and Marshall. *Book review in Bull. Amer. Math. Soc.* 44(2):267-276, 2007.

This is an expository survey of recent results in geometric function theory.

[51] C.J. Bishop. An A_1 weight not comparable to any quasiconformal Jacobian. *In the tradition of Ahlfors-Bers, IV*, volume 432 of *Contemp. Math.*, pages 7–18. Amer. Math. Soc., Providence, RI. 2007

Gives the example described by the title. This solved a problem of Stephen Semmes. Characterizing the Jacobians of quasiconformal maps has been one of the driving problems in the field, and this example shows that no sufficient condition only in terms of the distribution function is possible. I also construct a surface in Euclidean 3-space which is quasisymmetrically equivalent to the plane, but which is not bi-Lipschitz equivalent to the plane. No previous example was even embeddable in any Banach space.

[52] C.J. Bishop and H. Hakobyan. A central set of dimension 2. *Proc. Amer. Math. Soc.*, pages 2453-2461, 136(2008), no. 7.

The medial axis of a domain is the set of centers of disks in the domain which hit the boundary in two or more points. The central set is the set of centers of maximal disks in the domain. They are distinct sets in general, but are sometimes identified by mistake in the literature. Here we emphasize the difference by constructing a Lipschitz domain where the medial axis has dimension 1 and the central set has dimension 2.

[53] C.J. Bishop. Conformal welding and Koebe's theorem. *Ann. of Math.* 166(2): 613–656, 2007.

A conformal welding is a homeomorphism of the unit circle to itself of the form $h = g^{-1} \circ f$ where f and g are conformal maps of the two sides of the circle to two sides of a closed Jordan curve. It is known that not every homeomorphism has this form, but I prove that every homeomorphism is "almost" a conformal welding in a precise sense. This paper also contains a new and simple proof of the famous result that every quasisymmetric circle mapping is a conformal welding. It also contains a characterization of the weldings of "flexible curves"; the images of these curves under homeomorphisms conformal off the curve are dense in all closed curves.

[54] C.J. Bishop. Decreasing dilatations can increase dimension. *Proc. Amer. Math. Soc*, 136: 2453–2461, 2008.

Answers a question of Zinsmeister about quasiconformal maps and dimension.

[55] C.J. Bishop. A set containing rectifiable arcs locally but not globally. Pure and Applied Math. Quarterly, 7(1): 121-138, 2011. Special issue in honor of Fred Gehring, part 1 of 2.

I construct a compact set E of zero area and a $K > 1$ so that any K-quasiconformal image of E contains a rectifiable arc, but so that some quasiconformal image of E does not. This is related to the quasiconformal Jacobian problem, in that any function that blows up as it approaches E cannot be the Jacobian of any K-quasiconformal mapping.

[56] C.J. Bishop. Conformal mapping in linear time. Discrete and Computational Geometry, 44(2) 330-428, 2010.

Gives a linear time algorithm for computing conformal maps. The motivation for this method comes from hyperbolic manifolds and computational geometry. Moreover, the method is near optimal. It will find an ϵ approximation to the conformal map onto an n-gon in time $O(n|\log\epsilon\log\log\epsilon|)$. The techniques applied come from hyperbolic 3-manifolds, computational geometry, numerical analysis and geometric function theory. I would like to think this is one of my best results.

[57] C.J. Bishop. Bounds for the CRDT algorithm. Computational Methods in Function Theory, 10(1): 325-366, 2010.

In 1998 Driscoll and Vavasis introduced the CRDT algorithm for computing conformal maps. Although it works well in practice, there is no proof that it converges to the correct answer. We partially address this by showing the CRDT algorithm will always give an answer that is close to the correct answer in a certain precise sense (with bounds independent of the domain)

[58] C.J. Bishop. Optimal angle bounds for quadrilateral meshes. Discrete and Computational Geometry, 44(2): 308-329, 2010.

This shows that any polygon has a linear size quadrilateral mesh so that every new angle used is between 60 and 120 degrees. The angle bounds are optimal. This answers a question of Bern and Eppstein. Previously, it was not even known if there was a linear size mesh with all new angles bounded from below.

[59] C.J. Bishop. Tree-like decompositions and conformal maps. *Annals Acad. Sci. Fenn.*, 35(2): pages 389-404, 2010.

A companion paper showed that every planar simply connected domain has a collection of disjoint crosscuts (Jordan arcs with endpoints on the boundary) that divide the domain into uniformly chord-arc pieces. This paper uses such a decomposition to construct an approximation to the conformal map onto a disk that is within a uniformly bounded quasiconformal distance from the true map. Some examples of how this can be used to construct an iterative algorithm for conformal mapping are discussed.

[60] C.J. Bishop. A random walk in analysis. In the collection *All That Math: portraits of mathematicians as young readers*, 2011, a special volume of Revisita Matematica Iberoamericana, celebrating the Centennial of the Real Sociedad Matematica Espanola, Edited by Antonio Cordoba, Jose L. Fernandez and Pablo Fernandez

This is an essay written for a special issue of Revista Mat Iberoamericana describing how certain papers have influenced the course of my own work. Of particular importance were the

papers of Makarov on harmonic measure, Jones on rectifiable sets and Sullivan on hyperbolic convex hulls.

[61] C.J. Bishop. True trees are dense. *Invent. Mat.* 197(2): pages 433-452, 2014.

We show that any compact, connected set in the plane can be approximated by the critical points of a polynomial with only two critical values. This is related to the 'true form' of a finite tree in the plane, i.e., I show that such true forms are dense in all compact connected sets.

[62] C.J. Bishop with E. Feinberg and J. Zhang. Examples concerning Abel and Cesaro limits. *J. Math. Analysis and App.*, 420(2): pages 1654-1661, 2014.

This note describes examples of all possible equality and strict inequality relations between upper and lower Abel and Cesaro limits of sequences bounded above or below. It also provides applications to Markov Decision Processes. This paper has been cited in the Economics literature on optimal utility.

[63] C.J. Bishop. The order conjecture fails in S. *Journal d'Analyse*, 127(1): pages 283–302, 2015.

We show there is an entire function f with only four critical values and no finite asymptotic values whose order can change under a quasiconformal equivalence. This disproves the so called 'order conjecture' in the Speiser class. A counterexample in the Eremenko-Lyubich class had been independently found by Epstein and Rempe.

[64] C.J. Bishop. Constructing entire functions by quasiconformal folding. *Acta. Math.*, 214(1): pages 1–60, 2015.

We give a method for constructing transcendental entire functions with good control of both the singular values of f and the geometry of the tracts of f. The method consists of first building a quasiregular map by "gluing together" copies of the right half-plane that have each been quasiconformally "folded" into themselves. The measurable Riemann mapping theorem is then invoked to produce an entire function with similar geometry. Applications include constructing transcendental wandering domains and counterexamples (with only finite many singular values) to Wiman's conjecture, the area conjecture, the strong Eremenko conjecture and the order conjecture.

[65] C.J. Bishop and K. Pilgrim. Dynamical dessins are dense. *Rev. Mat. Iberoamericana*, 31(3): pages 1033-1040, 2015.

We show that every compact connected set in the plane can be approximated in the Hausdorff metric by the Julia set of a post-critically finite polynomial (and thus the Julia set is a dendrite). This uses the result from "True trees are dense" [61] that true trees approximate any continuum, combined with work of Pilgrim that every true tree is approximated by a Julia set of the type above.

[66] C.J. Bishop. Models for the Eremenko-Lyubich class *J. London Math. Soc.*, 92(1): 202-221, 2015.

This paper deals with approximation in a quasiconformal sense of models: a model domain is a union of disjoint simply connected domains and a model function is a conformal map of each connected component to the right half-plane followed by exponentiation. I show that any model can be approximated by models arising from Eremenko-Lyubich functions (entire functions with bounded singular sets). This reduces the construction of Eremenko-Lyubich entire functions with certain properties to the construction of models with these properties and this is often much easier.

[67] C.J. Bishop. Nonobtuse triangulations of PSLGs *Discrete and Computational Geometry*, 56(1): pages 43–92, 2016.

We show that any planar PSLG with n vertices has a conforming triangulation by $O(n^{2.5})$ nonobtuse triangles; they may be chosen to be all acute or all right. This is the first polynomial bound for nonobtuse triangulation of PSLGs. This result also improves a previous $O(n^3)$ bound of Eldesbrunner and Tan for conforming Delaunay triangulations. In the special case that the PSLG is the triangulation of a simple polygon, I show that only $O(n^2)$ elements are needed, improving an $O(n^4)$ bound of Bern and Eppstein. We also show that for any $\epsilon > 0$, every PSLG has a conforming triangulation with $O(n^2/\epsilon^2)$ elements and with all angles bounded above by $90° + \epsilon$. This improves a result of S. Mitchell when $\epsilon = \frac{3}{8}\pi = 67.5°$ and Tan when $\epsilon = \frac{7}{30}\pi = 42°$.

[68] C.J. Bishop. Quadrilateral meshes for PSLGs *Discrete and Computational Geometry*, 56(1): pages 1–42, 2016.

We prove that any PSLG has a conforming quadrilateral mesh with $O(n^2)$ elements and all new angles between $60°$ and $120°$ (the complexity and angle bounds are both sharp). Moreover, all but $O(n)$ of the angles may be taken in a smaller interval, say $[89°, 91°]$. This paper uses a result from [67], "Nonobtuse triangulation of PSLGs".

[69] C.J. Bishop, H. Hakobyan and M. Williams. Frequency of dimension distortion under quasisymmetric mappings , *Geometric and Functional Analysis (GAFA)*, 26(2): pages 379–421, 2016.

It is well known that a planar quasiconformal map f can send the segment $[0, 1]$ to a fractal curve of dimension d greater than 1, but it has been an open problem whether this can happen simultaneously for an uncountable set of parallel segments, i.e., can we increase the dimension of every component of $E \times [0, 1]$ for some uncountable E? It was even unknown whether one could make the image purely unrectifiable (no rectifiable subarcs). This paper proves that this is indeed possible and we give estimates for how much dimension can be increased. For each d between 0 and 1 we construct a set E of dimension d so that every component of $E \times [0, 1]$ is mapped to a curve of dimension as close to $2/(d + 1)$ as we wish, and we prove this upper bound is sharp. We also give a number of other results concerning purely unrectifiable images, sets whose dimension can't be lowered by QC maps, and dimension bounds in higher dimensions and for maps into metric spaces. The paper also proves that if E is Ahlfors regular in R^n and f is quasiconformal then $\dim(f(E+x)) = \dim(E)$ for Lebesgue almost every x. Indeed, this equality holds not just for Euclidean space, but on any Carnot group, and the inequality $\leq$ holds even more generally.

[70] C.J. Bishop. Models for the Speiser class. *Proc. London Math. Soc.*, 114(3), 765–797, 2017.

This is the Speiser class version of [66] dealing with the Eremenko-Lyubich class. Both papers deal with approximation in a QC sense of models: a model domain is a union of disjoint simply connected domains and a model function is a conformal map of each connected component to the right half-plane followed by exponentiation. In this paper I show that any model function can be approximated by models arising from the Speiser class, but the Speiser model domain many have more tracts than the target being approximated; this is unavoidable in some cases. However, I prove the sharp result that at most twice as many tracts are needed for the approximation. I also give some results that place geometric restrictions on Speiser models that distinguish them from Eremenko-Lyubich models.

[71] C.J. Bishop. A transcendental Julia set of dimension 1. Inventiones Math., 212(2), 407–460, 2018.

In 1975 Baker proved that if f is a transcendental entire function, then Julia set of f contains a continuum and hence has Hausdorff dimension at least 1. Misiurewicz and McMullen had each given examples showing dimension 2 is possible and Stallard showed every value in (1,2] is possible, but the question of whether dimension 1 can be attained has remained open. I show the answer is yes in the strongest possible way by showing there is a transcendental entire function whose Julia set has finite length on the sphere. The packing dimension of this example is also 1, the first known example where the dimension is less than 2. This example does not lie on a rectifiable curve on the sphere; whether this is possible remains open.

[72] C.J. Bishop and C. LeBrun. Anti-Self-Dual 4-manifolds, Quasi-Fuchsian groups and almost-Kahler geometry *Communications in Analysis and Geometry*, special issue dedicated to Karen Uhlenbeck, 28(4), 745–780, 2020.

This paper gives an application of harmonic measure in three dimensions to the geometry of 4-manifolds. The analytic part of the paper is the construction of a closed Jordan curve in the plane that is the limit set of finitely generated, co-compact Fuchsian group and so that the harmonic measure of one side of the curve defines a hyperbolically harmonic function in the upper half-space that has a critical point. From the existence of this harmonic function, we deduce the existence of a compact Riemannian 4-manifold that is anti-self-dual but not almost-Kahler. The example is interesting because the almost-Kahler metrics form a non-empty, open subset of the anti-self-dual metrics on such a manifold, and this example shows, for this first time, that this can be a proper open subset. A more careful argument (proving that the critical point is non-degenerate, hence persists under small perturbations) shows that the complement (the non-almost-Kahler metrics) also has interior, so it would be interesting in the future to investigate the properties of the boundary between these regions.

[73] C.J. Bishop and K. Lazebnik. Prescribing the Postsingular Dynamics of Meromorphic Functions, to appear in *Math. Annalen*, 375(3), 1761-1782, 2019.

We show that any dynamics on any discrete planar sequence can be realized by the postsingular dynamics of some transcendental meromorphic function, provided we allow for small perturbations of the set. This work is motivated by an analogous result of DeMarco, Koch and McMullen in for rational functions. The proof contains a variation of quasiconformal folding for constructing meromorphic functions and an argument of the second author involving a infinite dimensional fixed point theorem.

[74] C.J. Bishop. A Speiser class Julia set with dimension near one, with S. Albrecht. *Journal d'Analyse*, special issue dedicated to Larry Zalcman, 141(1), 49–98, 2020.

In 1975 Baker proved that the Julia set of a non-polynomial (transcendental) entire function contains a non-trivial continuum, and hence has Hausdorff dimension at least 1. Stallard gave examples with all dimensions strictly between 1 and 2, and I constructed a transcendental entire function whose Julia set has dimension equal to 1. Stallard's examples are in the Eremenko-Lyubich class, that is, their singular sets are bounded, and she proved that the Julia set of any EL function has dimension strictly larger than 1. In this paper Albrecht and I construct functions in the Speiser class (finite singular sets) whose Julia sets have dimension as close to 1 as desired, strengthening Stallard's examples in the EL class. The construction is an application of my quasiconformal folding method, perhaps the most intricate application to date.

[75] C.J. Bishop, H. Drillick and D. Ntalampekos. Falconers' distance set conjecture can fail for strictly convex sets in $\mathbb{R}^d$. *Revista Mat. Iberoamericana*, 37(5), 1953–1968, 2021.

For any norm on $\mathbb{R}^d$ with countably many extreme points, there is a compact set E whose distance set with respect to this norm has zero linear measure. This was previously known only for norms associated to certain finite polygons in the plane. Similar examples exist for norms that are very well approximated by polyhedral norms, including some examples where the unit ball is strictly convex and has C^1 boundary. This paper corrects a gap in a paper of Falconer noticed by the second author while writing her undergraduate honors thesis at Stony Brook.

[76] C.J. Bishop. Quasiconformal maps with thin dilatations. *Publicacions Matemàtiques* vol 66(2022), 715-727.

We give an estimate that quantifies the fact that a normalized quasiconformal map whose dilatation is non-zero only on a set of small area approximates the identity on the whole plane.

[77] C.J. Bishop. Conformal images of Carleson curves. *Proc. Amer. Math. Soc.* 9 (2022), 90–94.

We show that arclength on curve inside the unit disk is a Carleson measure iff the image of the curve has finite length under every conformal map onto a domain with rectifiable boundary. This answers a question posed by Percy Deift.

[78] C.J. Bishop. Uniformly acute triangulations of PSLGs *Discrete & Computational Geometry* 70(2023), 1090–1120.

We answer a question of Florestan Brunck by showing that any planar triangulation that has a lower angle bound θ, has an acute triangulation that with an upper angle bound strictly below 90° and depending only on θ. More generally, there is a universal $\theta_0 > 0$ so that any PSLG has with minimal interior angle θ has a conforming acute triangulation with all angles in $[\theta_0, 90^\circ - \theta_0]$, except for triangles touching a vertex v of the PSLG where the interior angle is $\theta_v < \theta$; these are all isosceles triangles with angles in $[\theta_v, 90^\circ - \theta_v/2]$.

[79] C.J. Bishop. Uniformly acute triangulations of polygons. to appear *Discrete Comput. Geom.*

This improves the constant in the previous paper in the case that the PSLG is a simple polygon. Every P has an acute triangulation with all angles in $[30^\circ, 75^\circ]$, except for triangles that contain a vertex x where P has interior angle is $\theta_v < 30^\circ$; these are all isosceles triangles with angles in $[\theta_v, 90^\circ - \theta_v/2]$.

[80] C.J. Bishop. The traveling salesman theorem for Jordan curves. *Advances in Math.* 404 (2022), part A, Paper No. 108443, 27 pp.

This gives an improvement of Peter Jones's traveling salesman theorem that holds for Jordan curves, but not for general sets. His theorem implies that the length of a Jordan arc Γ is bounded by $1 + \delta$ times the diameter of Γ plus a sum of β-numbers. This paper shows that the diameter term can be replaced by the chord-length of Γ, i.e., the distance between its endpoints. This is true in all finite dimensions (with a dimension dependent constant). A corollary of our self-contained argument proves the usual TST in all dimensions (a result of Okikiolu). The sharper version of TST is used in the paper " Weil-Petersson curves, Möbius energies, traveling salesman theorems and minimal surfaces".

[81] C.J. Bishop, K. Lazebnik and M. Urbanski. Equilateral triangulations and the postcritical dynamics of meromorphic functions. to appear *Math. Annalen.*

We show that any dynamics on any planar set S, discrete in some domain D, can be realized by the postcritical dynamics of a function holomorphic in D, up to a small perturbation. A key step in the proof, and a result of independent interest, is that any planar domain D can be equilaterally triangulated with triangles whose diameters tend to 0 at any prescribed rate near the boundary. When D is the whole plane, the dynamical result was proved in my paper "Prescribing the Postsingular Dynamics of Meromorphic Functions", with Lazebnik by a different method (QC folding).

[82] C.J. Bishop. Function theoretic characterizations of Weil-Petersson curves. to appear *Revista Mat. Iberoamericana.*

This is a companion to the paper "Weil-Petersson curves, conformal energies, beta-numbers, and minimal surfaces". That paper gives various new geometric characterizations of Weil-Petersson in the plane that can be extended to curves in all finite dimensional Euclidean spaces. This paper deals with the 2-dimensional case, giving new proofs of some known characterizations, and giving new results for the conformal weldings of Weil-Petersson curves and a geometric characterization of these curves in terms of Peter Jones's beta-numbers.

PREPRINTS

[83] C.J. Bishop. Non-removable sets for quasiconformal and quasi-isometric mappings in $\mathbb{R}^3$.

We construct a Cantor set in 3-space which is not removable for QC mappings. This is the only known non-trivial example of a non-removable set in dimension higher than 2.

[84] C.J. Bishop. Interpolating sequences for the Dirichlet space and its multipliers.

We give a description of the universal interpolating sequences for the Dirichlet space and give partial results for the usual interpolating sequences for the Dirichlet space and for the space of its multipliers. This preprint was never published because it has significant overlap with independent work of Marshall and Sundberg, but has been cited numerous times.

[85] C.J. Bishop. Distortion of disks by conformal maps.

Answers a question of Astala about how a collection of disks can be distorted by a QC map which is conformal outside the disks.

[86] C.J. Bishop. A fast quasiconformal mapping theorem for polygons.

Gives a linear time approximation to the Riemann map. This is an introduction to the longer paper "Conformal mapping in linear time", [56].

[87] C.J. Bishop. Estimates for harmonic conjugation.

Gives a geometric characterization of the planar domains for which harmonic conjugation defines a L^2 bounded operator on the boundary. The answer is in terms on a decomposition of the domain into pieces and roughly says that conjugation is bounded iff the Poincaré inequality holds on a weighted tree related to the decomposition.

[88] C.J. Bishop. Another Besicovitch-Kakeya set. We give a 1-page proof that there is a planar set of zero area that contains a unit segment in every direction. Moreover, I show this set has near optimal Minkowski dimension.

[89] C.J. Bishop. A curve with no simple crossings by segments.

We construct a closed Jordan curve in plane that has an uncountable intersection with any closed line segment whose endpoints are in different complementary components of the curve. This answers a question posed to me by Percy Deift. Some additional questions are listed at the end of the note.

[90] C.J. Bishop. Weil-Petersson curves, β-numbers, minimal surfaces.

The Weil-Petersson class is collection of rectifiable planar quasicircles defined by Takhtajan and Teo's work and motivated by problems arising in string theory (theses curves have also been studied in Mumford's work on computer vision and are connected to Schramm-Loewner Evolutions in probability). All previously known characterizations of this class of curves are in terms of function theory, e.g., in terms of the conformal maps associated to the curve, and it has been an open problem to give an intrinsic geometric characterization. This paper gives several such characterizations in terms Sobolev spaces, knot energies, rates of approximation by polygons, of Peter Jones' beta-numbers, hyperbolic convex hulls, minimal surfaces in hyperbolic space and renormalized area of such surfaces. Moreover, many of these characterizations extend to higher dimensions and remain equivalent there.

[91] C.J. Bishop. Conformal removability is hard.

We prove that the collection of compact planar sets that are non-removable for conformal homeomorphisms is not a Borel subset of the space of all compact subsets with the Hausdorff metric. This contrasts with the collection of non-removable sets for bounded holomorphic functions, which is Borel. This paper is mostly a survey of the necessary facts from complex analysis and descriptive set theory needed to prove these claims, although a few new results are given and numerous questions posed.

[92] C.J. Bishop. BiLipschitz homogeneous hyperbolic nets.

This answers a question of Itai Benjamini by showing there is a value $K > 1$ so that for any $r > 0$, there exist r-dense discrete sets in the hyperbolic disk that are homogeneous with respect to K-biLipschitz maps of the disk to itself. However, this is not true for K close to 1; in that case, every K-biLipschitz homogeneous discrete set must omit a disk of hyperbolic radius $r(K) > 0$, depending only on K. For $K = 1$, this is a consequence of the Margulis lemma for discrete groups of hyperbolic isometries.

[93] C.J. Bishop and L. Rempe. Non-compact Riemann surfaces are equilaterally triangulable.

We show that every open Riemann surface can be obtained by gluing together a countable collection of equilateral triangles in such a way that every vertex belongs to finitely many triangles. Equivalently, every open surface supports a Belyi function, a holomorphic branched covering to the Riemann sphere that is only branched over $0, 1, \infty$. For compact surfaces this only occurs for algebraic curves by Belyi's theorem. Among the consequences is that every Riemann surface is a branched cover of the Riemann sphere, ramified over finitely many points. The result also constructs many new holomorphic dynamical systems of finite type on non-simply connected Riemann surfaces (generalizing rational dynamics on the sphere and transcendental dynamics on the plane).

[94] C.J. Bishop. Optimal triangulations of polygons.

It is a problem of long-standing interest to triangulate a polygon with the best possible bounds on the angles used. For any simple polygon P we compute the optimal upper and lower angle bounds for triangulating P with Steiner points, and show that these bounds are attained (except in one special case). The bounds for an N-gon are computable in time O(N); this adds to a short list of optimization problems that are faster to solve with Steiner points than without them. In general, we show the sharp upper and lower bounds cannot both be attained by a single triangulation, although this does happen in some cases. For example, we show that any polygon with minimal interior angle θ has a triangulation with all angles in the interval $I = [\theta, 90° - \min(36°, \theta)/2]$, and for $\theta \leq 36°$ both bounds are best possible. Surprisingly, we prove the optimal angle bounds for polygonal triangulations are the same as for triangular dissections. The proof of this verifies, in a stronger form, a 1984 conjecture of Gerver

[95] C.J. Bishop. Wandering domains.

We give a self-contained proof of Sullivan's no wandering domains theorem for polynomials, proving everything that is not usually found a in first year graduate course in real analysis, complex analysis or topology. We avoid the use of singular integrals to prove differentiable dependence of a quasiconformal map on its dilatation, using only continuous dependence. We pay for the simpler analysis by using a more sophisticated topological result: any continuous map from dimension n+1 to dimension n maps some non-trivial continuum to a point. The final part of the paper sketches what is currently known about wandering domains for entire functions.

[96] C.J. Bishop. Equi-triangulation of polygons.

We prove that any two polygons of the same area can be triangulated using the same set of triangles. This strengthens the 200-year-old Wallace–Bolyai–Gerwien theorem from dissections to triangulations.

[97] C.J. Bishop and K. Lazebnik. A geometric approach to polynomial and rational approximation.

We strengthen the classical approximation theorems of Weierstrass, Runge and Mergelyan by showing the polynomial and rational approximants can be taken to satisfy several additional geometric properties. In particular, when approximating a function f on a compact set K, all the critical points of our approximants lie close to K, and all the critical values lie close to $f(K)$. The proofs rely on extensions of (1) the quasiconformal folding method of the first author, and (2) a theorem of Caratheodory on approximation of bounded analytic functions by finite Blaschke products.

[98] C.J. Bishop and K. Lazebnik. Hilbert's lemniscate theorem for rational functions.

In 1897 Hilbert proved that any Jordan curve in the complex plane can be approximated in a strong sense by a polynomial lemniscate (a level curve of —p— for some polynomial p). We extend this by showing that any finite collection of N pairwise disjoint Jordan curves can be similarly approximated by a rational lemniscate. The number of poles needed is at most (N+1)/2; three can be specified exactly (up to some necessary topological restraints) and the remainder can be specified with arbitrary precision.

[99] C.J. Bishop and D.L. Bishop. Approximation by singular polynomial sequences.

This is a sequel to a paper by D.L. Bishop strengthening Weierstrass's theorem by showing any continuous function on [0,1] can be approximated by a polynomial with all critical points inside this interval, a question arising from complex dynamics. A result of Clunie and Kuijlaars says that (except is very special cases when the limit is a certain kind of entire function) the derivatives of such approximants cannot converge on a set of positive measure to finite, non-zero values. Thus almost everywhere they must either diverge or converge to either 0 or ∞. We show that all three possibilities can actually occur, and such approximants can be realized by perturbing the roots of certain Chebyshev polynomials.

BOOKS

[100] C.J. Bishop and Yuval Peres. Fractals in Analysis and Probability, Cambridge University Press, 2017

This is an introduction to Hausdorff, Minkowski and packing dimension, illustrated by examples drawn from analysis, dynamics and probability theory. It covers topics including self-similar sets, self-affine sets, the Weierstrass nowhere differentiable function, Brownian motion, Markov chains, Kakeya sets and Peter Jones' traveling salesman theorem.

[101] C.J. Bishop and Yuval Peres. Conformal Fractals (in preparation)

In the same spirit as the book above, but dealing with fractals that arise from conformal dynamics or involve conformal invariance. Currently planned chapters include: introduction to conformal invariants, diffusion limited aggregation, Makarov's theorems on planar, simply connected harmonic measure, the Jones-Wolff theorem on multiply connected domains, Wolff snowflakes, Julia sets, Kleinian limit sets, the Gaussian free field.

[102] C.J. Bishop. The Riemann Mapping Theorem (in preparation)

This gives an introduction to conformal mapping with an emphasis on various methods for numerically computing such maps.

[103] C.J. Bishop. Introduction to Transcendental Dynamics (in preparation)

This is an introduction to the iteration theory of entire functions and also includes introduction to the basic techniques of geometric function theory needed to understand them, such as hyperbolic geometry, normal families, potential theory, extremal length and quasiconformal maps in the plane.

[104] C.J. Bishop. Quasiconformal Mappings (in preparation)

A self-contained introduction to planar quasiconformal mappings up through the proof of the measurable Riemann mapping theorem and some of its applications to rational and transcendental dynamics.

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