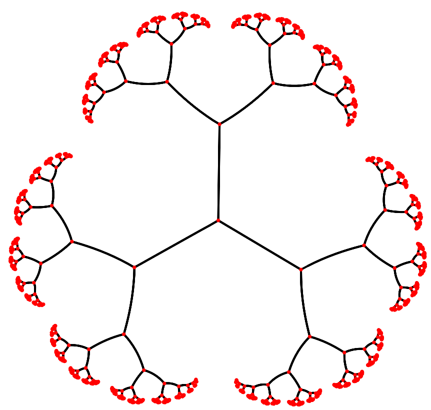
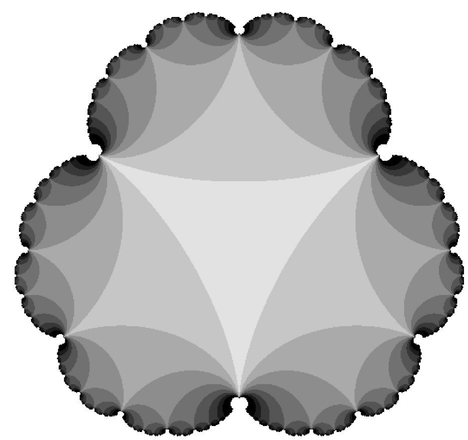
MAT 670, Fall 2023, Stony Brook University

TOPICS IN COMPLEX ANALYSIS
DESSINS AND DYNAMICS
Remarks on binary tree and type problem

Christopher Bishop, Stony Brook





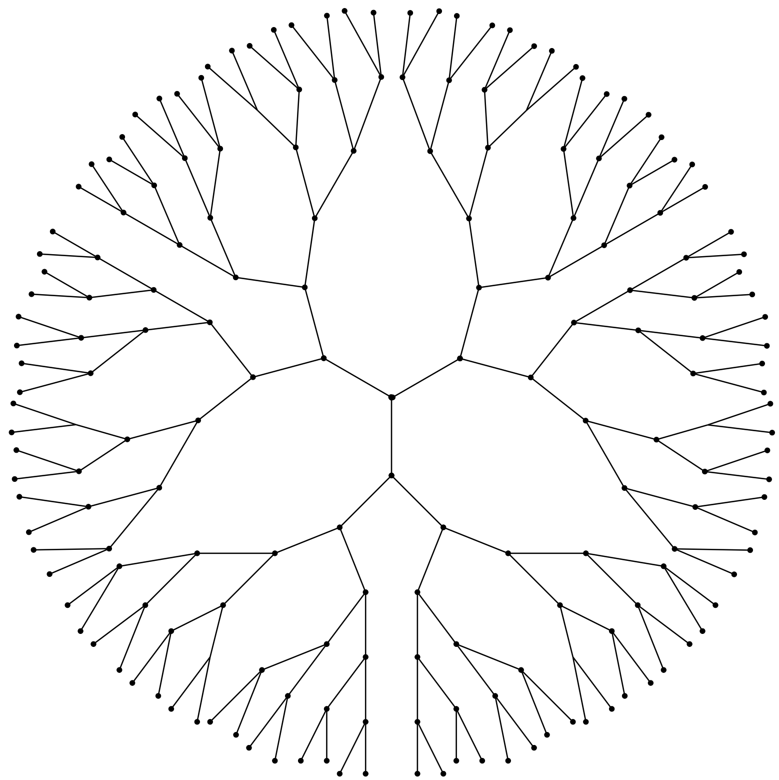
Given a meromorphic function f, a point $a \in \mathbb{C}$ is said to be a totally ramified value (of multiplicity $m \geq 2$) if all preimages of a under f have multiplicity at least m.

Take $m = \infty$ if a is an omitted value (e.g. 0 for e^z or ∞ for an entire function) .

Theorem (Nevalinna): If f is entire and has q totally ramified values of multiplicities m_k for $k = 1, \ldots, q$ then

$$\sum_{k=1}^{q} (1 - \frac{1}{m_k}) \leq 2.$$

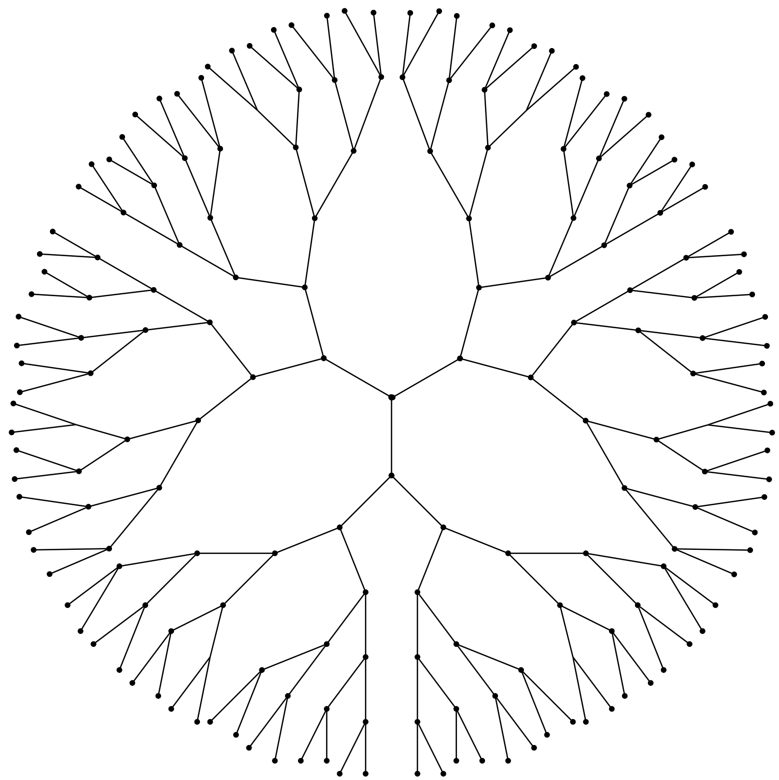
The infinite 3-regular tree has no true form in plane.



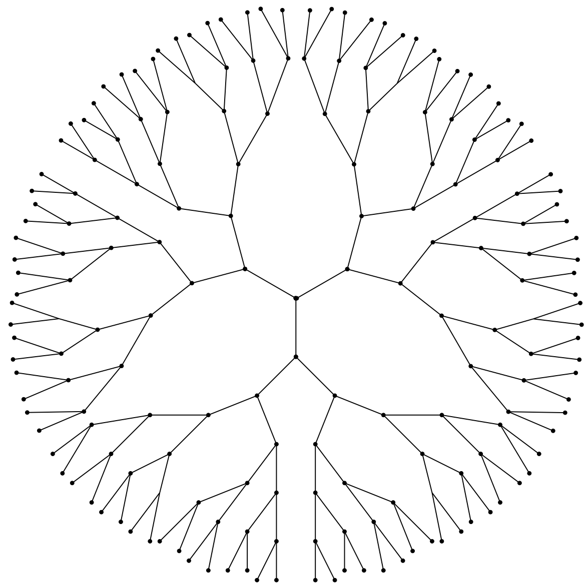
Reason 1 (W. Cui, 2018): Corresponding f contradicts Nevanlinna's theorem $\sum_{i=1}^{q}(1-\frac{1}{m_i}) \leq 2$, $q=3, m_1=m_2=3, m_3=\infty$.

Cui also gives sufficient conditions for an infinite tree to have a true form.

The infinite 3-regular tree has no true form in plane.

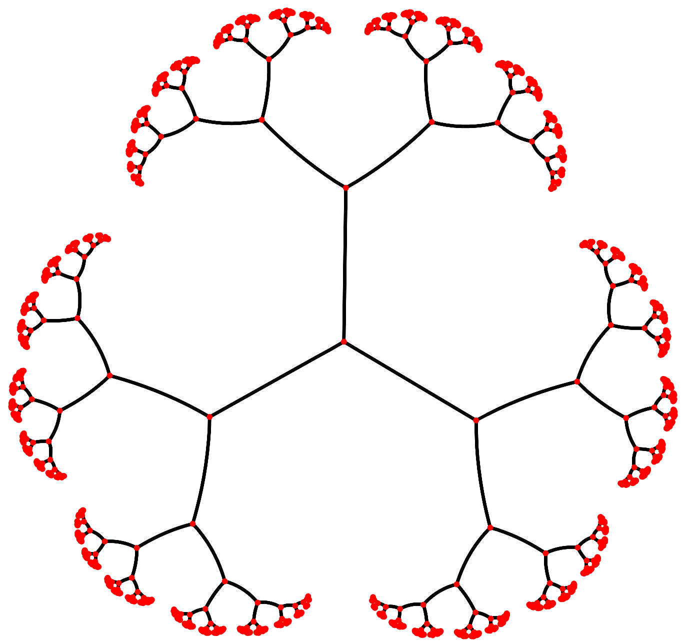


Reason 2: Planar homeomorphisms mapping tree to itself permute the complementary components. Corresponding conformal maps are continuous across tree, hence linear, hence isometries. But this group has exponential growth: “too large” to fit inside isometries of plane.

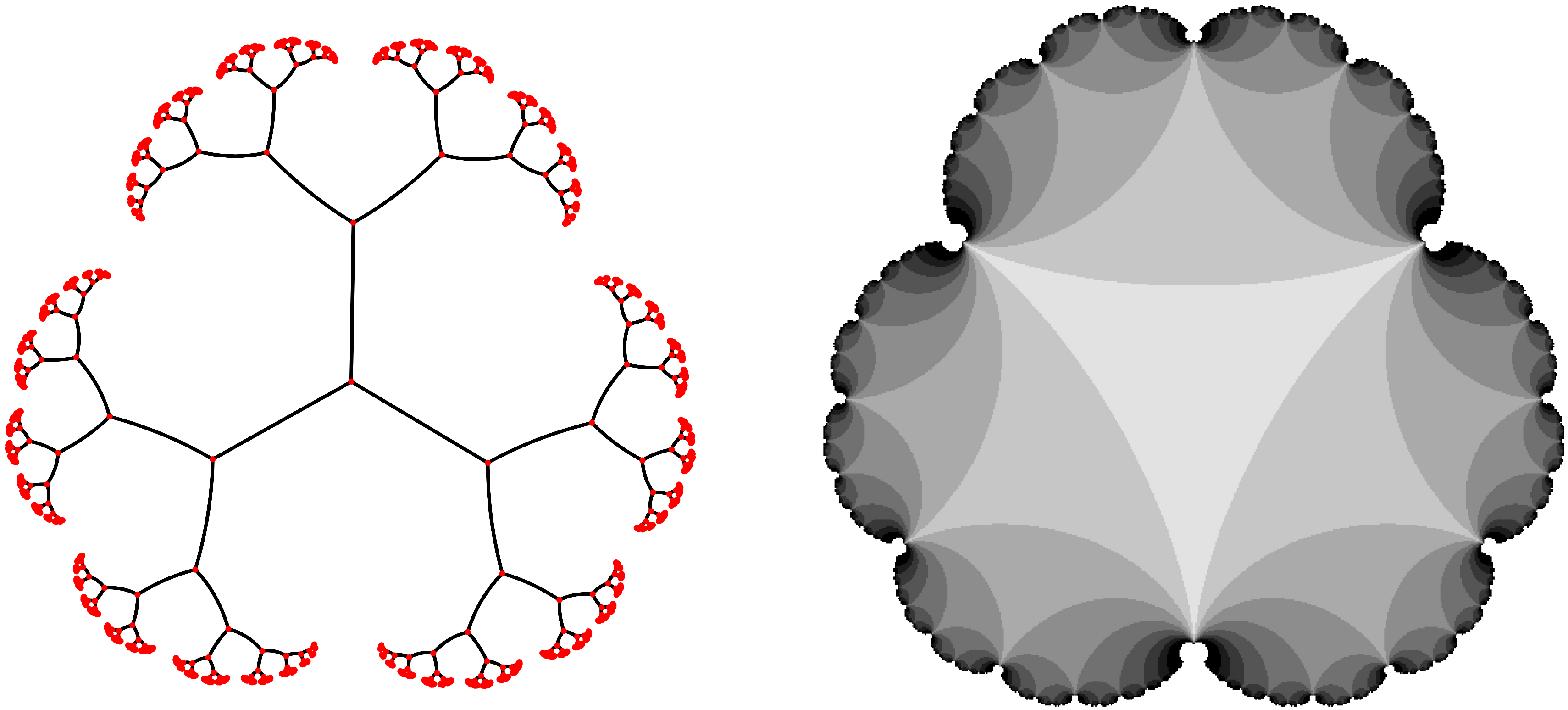


- If so, tree automorphisms permute complementary components.
- Corresponding conformal maps extend continuously across tree.
- Morera's theorem implies these give conformal linear maps.
- Thus vertices are ϵ-separated, but 3^n occur within $O(n)$ of origin.
- Impossible in plane.

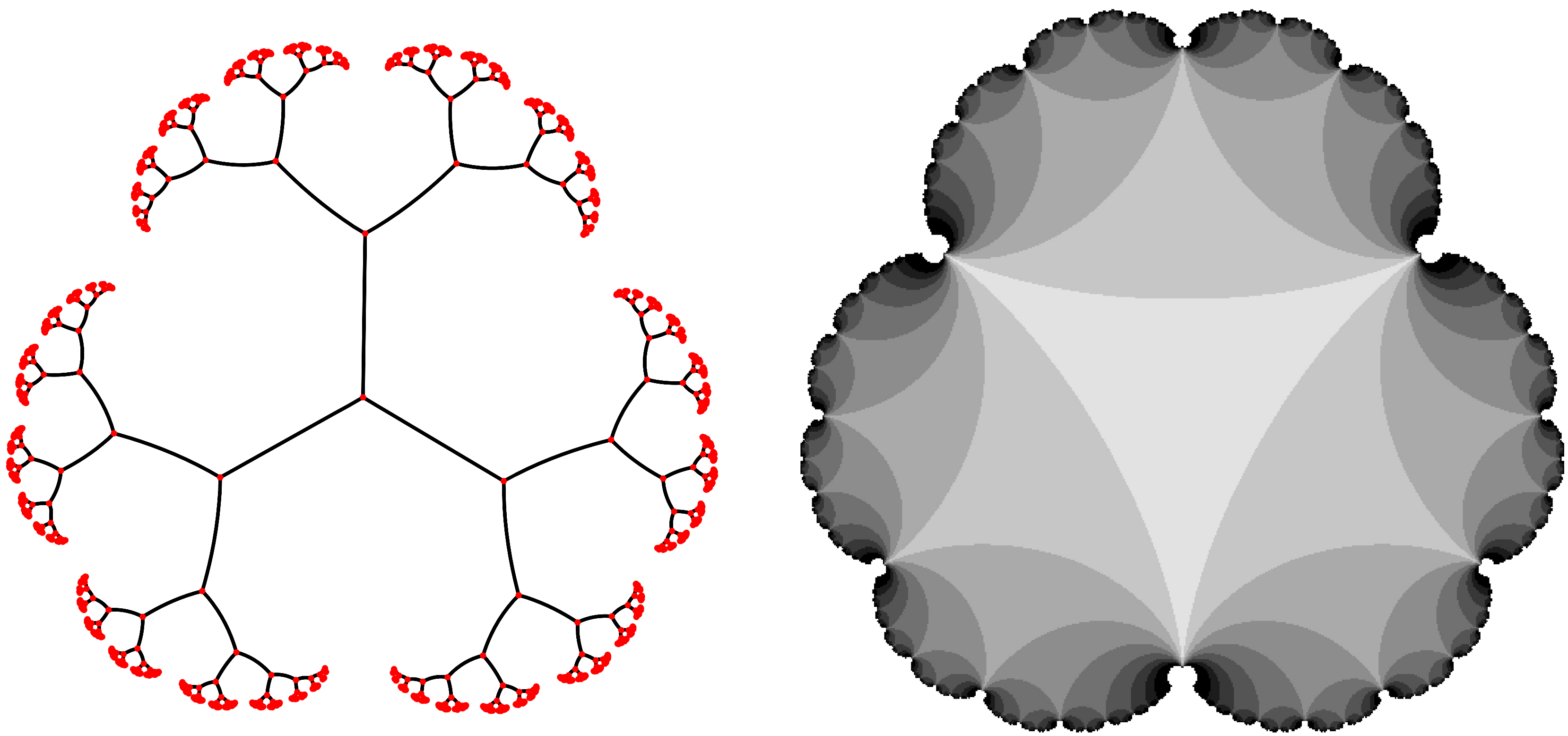
The infinite 3-regular tree has no true form in plane.



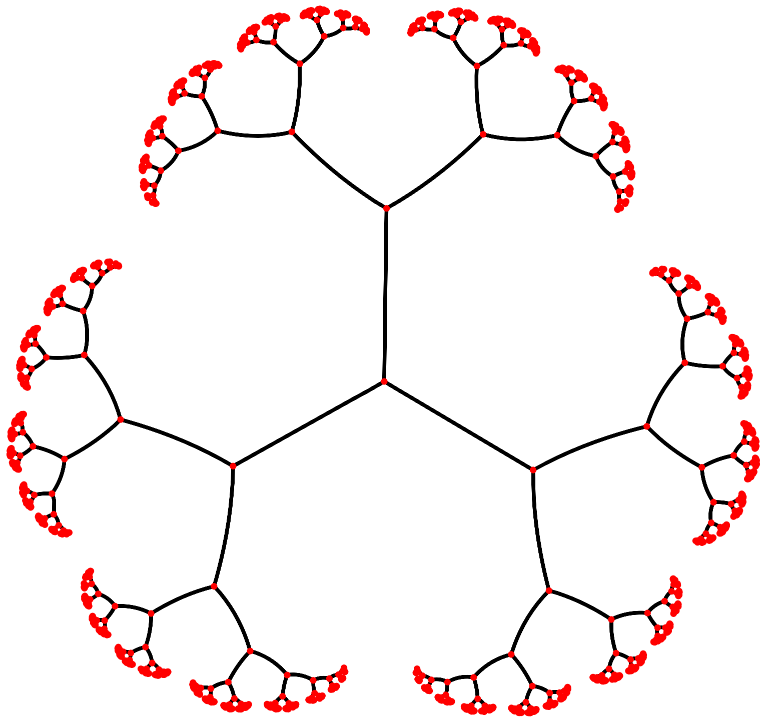
"Reason" 3: Computation of true form of truncated regular tree indicate convergence to bounded set. (Image due to Marshall and Rohde.)

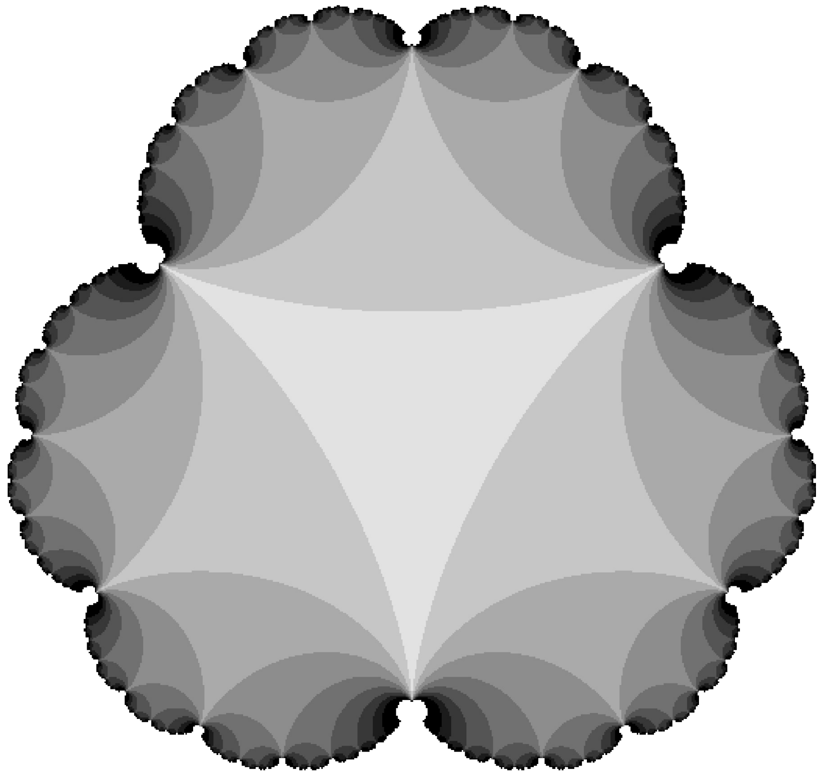


Coincidence of shapes noted by Rohde and Werness.
Deltoid fractal studied by Lee, Lyubich, Makarov, Mukherjee.
Arises in anti-holomorphic dynamics.

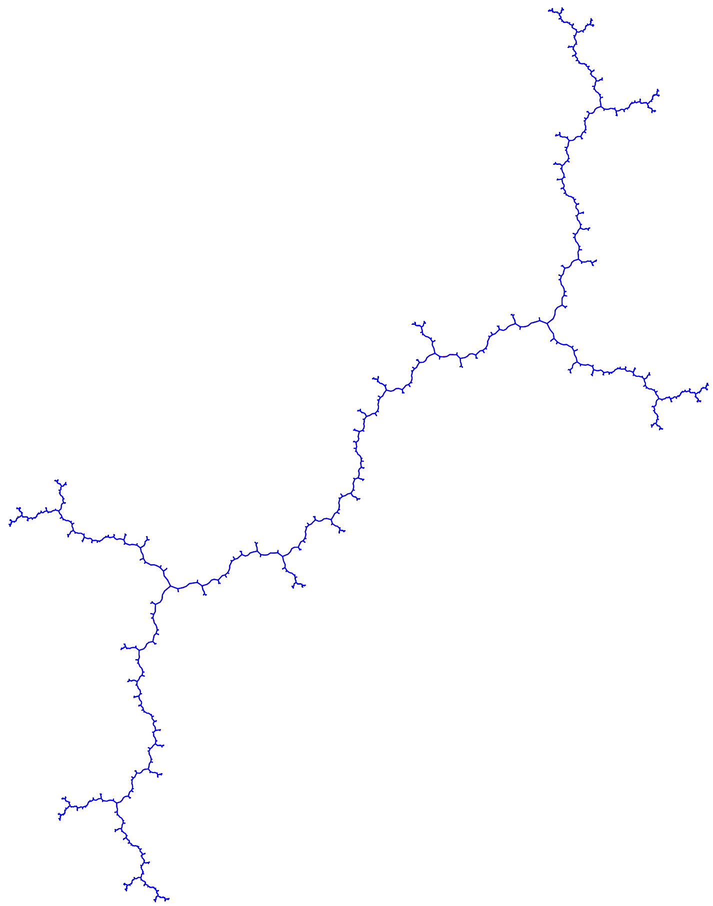


True form of truncated finite 3-regular tree,
and limit set of reflected deltoid group.

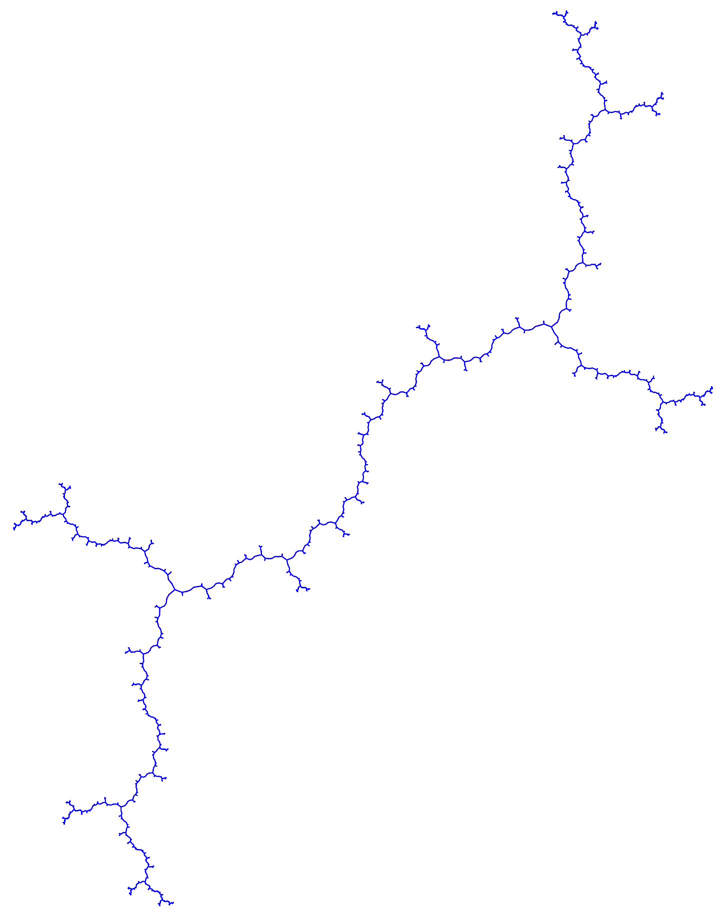




Proved equal by Oleg Ivrii, Peter Lin, Steffen Rohde and Emanuel Sygal
Related to matings of Julia sets with Kleinian groups,
studied by Lee, Lyubich, Makarov and Mukherjee.



This is not the Julia set of $z^2 + c$, $c \approx 0.288 + 1.115i$.
(But it looks exactly like it.)



Edges not all equal harmonic measure, but still identified by conformal map.
Rigidity: combinatorial data determines geometry.