Take Home Final, MAT 551, due at my office, Thurs., Dec. 13, 2012

Name

Do 15 of the following (your choice) :

(1) *Chapter 15, Exercise 2, page 164:* Let X and U be Banach spaces, U reflexive. Let $\mathbf{M}$ be a bounded linear map $X \to U$. Let x_n be a sequence in X weakly convergent to x. Then $\mathbf{M}x_n$ converges weakly to $\mathbf{M}x$.

(2) *Chapter 15, Exercise 11, page 171:* Prove Theorem 14: X and U are Banach spaces, $\mathbf{M} : X \to U$ a bounded linear map. Assume that the range of M has finite codimension in U and prove the range is closed.

(3) *Chapter 15, Exercise 13, page 172:* Prove Theorem 16: X and U are Banach spaces, $\mathbf{M} : X \to U$ a bounded linear map whose range is a subset of U of second category. Then the range of $\mathbf{M}$ is all of U.

(4) *Chapter 17, Exercise 1, page 201:* Show that if $\mathbf{P}$ is a non-zero projection, that is, satisfies $\mathbf{P}^2 = P \neq 0$, then $|P| > 1$.

(5) *Chapter 17, Exercise 2, page 201:* Show that the spectral radius $|\sigma(\mathbf{M})|$ depends upper semicontinuously on $\mathbf{M}$ in the norm topology, namely, if $\lim_n \mathbf{M}_n = \mathbf{M}$, then $\limsup |\sigma(\mathbf{M})| \leq |\sigma(\mathbf{M})|$.

(6) *Chapter 17, Exercise 3, page 201:* Show that $|\exp(\mathbf{M})| \leq \exp(|\mathbf{M}|)$.

(7) *Chapter 19, Exercise 1, page 210:* Prove Theorem 1: Suppose S is a compact Hausdorff space. Every maximal ideal in $C(S)$ is of the form $\{f : f(r) = 0\}$ for some $r \in S$.

(8) *Chapter 19, Exercise 5, page 221:* Let f be a function in L^2 on $\mathbb{R}$. Show that the translates of f span L^2 if and only if the Fourier transform of f does not vanish on a set of positive Lebesgue measure.

(9) *Chapter 21, Exercise 1, page 233:* Prove the following statements about precompact sets in a Banach space:

  (a) If C_1, C_2 are precompact, so is $C_1 + C_2$.

  (b) If C is precompact, so if its convex hull.

  (c) If C is precompact in X and $\mathbf{M} : X \to U$ is a linear, bounded map to a Banach space U, then $\mathbf{M}C$ is precompact in U.

(10) *Chapter 21, Exercise 5, page 241:* Show that a compact operator on an infinite dimensional Banach space is not invertible.

(11) *Chapter 21, Exercise 8, page 243:* Show by example that the strong limit of a sequence of compact operators need not be compact.

(12) *Chapter 25, Exercise 1, page 275:* Prove that if $\mathbf{M} : X \to X$ is invertible and Y is a finite dimensional subspace of X invariant under $\mathbf{M}$, then M is invertible on Y and on X/Y.

(13) *Chapter 25, Exercise 2, page 275:* Show that the subspaces of X invariant under an operator M for a lattice in the sense that the intersection of any two invariant subspaces is invariant and the closure of the sum of two invariant subspaces is invariant.

(14) *Chapter 28, Exercise 5, page 318:* Show that if x is the weak limit of x_n and $\lim_n \|x\| = \|x\|$ then x_n converges strongly to x.

(15) *Chapter 28, Exercise 7, page 320:* Show that the positive square root of a positive, compact symmetric operator is unique.

(16) *Chapter 30, Exercise 2, page 330:* Show that $\|T\| \leq \|T\|_{\mathrm{tr}}$.

(17) *Chapter 31, Exercise 1, page 330:* Show that
   (a) The inverse of an invertible symmetric operator is symmetric.
   (b) The product of symmetric operators is symmetric.
   (c) The set of symmetric operators is closed in the weak topology of operators.

(18) *Chapter 31, Exercise 8, page 330:* Show that a unitary operator preserves inner products.

(19) *Chapter 31, Exercise 9, page 330:* Show that $U^* = U^{-1}$ implies U is unitary.

(20) *Chapter 31, Exercise 10, page 330:* Show that the spectrum of a unitary operator lies on the unit circle.