MAT 487 Fall 2013, Tutorial on Analysis, Midterm

Name	ID

Problem 1 (10 points): Give the correct definition or statement.

(1) State the five axioms for addition.

(2) State the Schwarz inequality.

(3) Define an equivalence relation.

(4) Define a metric.

(5) Define open set.

(6) Define perfect set.

(7) Define compact set.

(8) Define connected set.

(9) Define Cauchy sequence.

(10) Define a complete metric space.

Problem 2 (10 points): Give an example of each, or explain why it can't exist:

(1) A subset of $\mathbb{R}$ that is both open and closed.

(2) A divergent series $\sum a_n$ so that $\sum a_n^2$ converges.

(3) A compact set with no limit points.

(4) A closed subset of $\mathbb{R}$ that is not compact.

(5) A convergent series $\sum a_n$ so that $\sum (-1)^n a_n$ diverges.

(6) A monotome function on $\mathbb{R}$ that is discontinuous at every rational number.

(7) Sequence of nested closed sets $E_1 \supset E_2 \supset E_3 \cdots$ whose intersection is empty.

(8) A bounded function on $\mathbb{R}$ that does not attain its supremum.

(9) A countable set that is dense in $\mathbb{R}$.

(10) A 1-to-1, onto mapping beween $(0, 1)$ and $[0, 1]$.

Problem 3 (10 points): Give a proof of two of the folllowing statements.

(1) If $F : X \to Y$ is a continuous map between metric spaces and $K \subset X$ is compact, then $f(K)$ is also compact.

(2) If $\{a_n\} > 0$ and $\sum a_n$ converge, then $\sum a_n^2$ also converges.

(3) The set of convergent, integer valued sequences is countable.

(4) If X is a metric space, $E \subset X$ is closed and $F \subset X$ is compact prove that

$$\inf\{d(x,y) : x \in E, y \in Y\} > 0.$$

Show this can fail if F is only closed.

(5) Show that there is no continuous, 1-to-1, onto map between $(0,1)$ and $[0,1]$.