MAT 487 Fall 2013, Tutorial on Analysis, Final

Name	ID

Problem 1 (10 points): Give the correct definition or statement.

(1) Define the derivative of a function f at a point x.

(2) Define local maximum.

(3) State the generalized mean value theorem.

(4) State Taylor's theorem.

(5) Define partition.

(6) Define a common refinement of two partitions.

(7) Define the Riemann-Stieltjes integral $\int f d\alpha$.

(8) State the fundamental theorem of calculus.

(9) Define a curve in $\mathbb{R}^d$.

(10) Define the length of a curve.

Problem 2 (10 points): Give an example of each, or explain why it can't exist:

(1) A function f differentiable at 0, but not continuous at zero.

(2) A continuous function f on the reals that is not differentiable at 0.

(3) A function f differentiable and continuous at zero, but not continuous anywhere else.

(4) A continuous and differentiable function f on the whole real line so that f' is not continuous at 0.

(5) An increasing function that has a negative derivative at 0.

(6) An increasing, continuous function that is not differentiable at infinitely many points.

(7) A function on $[0, 1]$ that is not Riemann integrable.

(8) A function on $[0, 1]$ that has infinitely many discontinuities, but is Riemann integrable.

(9) A sequence of Riemann integrable functions that converges at every x to a function that is not Riemann integrable.

(10) A curve in $\mathbb{R}^2$ that is not rectifiable.

Problem 3 (5 points): Give a proof of one the following statements.

(1) If f is increasing and bounded on $[0,1]$ then it is Riemann integrable.

(2) Suppose $f \geq 0$ and is continuous on $[0,1]$. If $\int_1^b f dx = 0$ for all $0 \leq a < b \leq 1$, prove that $f(x) = 0$ for all $x \in [0,1]$.

(3) Prove that if $\{f_n\}$ are Riemann integrable functions on $[0,1]$ that converge uniformly to a function f, then f is also Riemann integrable on $[0,1]$.

Problem 4 (5 points): Give a proof of one the following statements.

(1) Prove that there is an infinitely differentiable function f that is zero for $x \leq 0$ and positive for $x > 0$.

(2) Prove that for every integer $n > 0$ there is a polynomial p of degree n so that

$$\max_{x \in [0,1]} |e^x - p(x)| \leq \frac{e}{n!}.$$

(3) Suppose f is infinitely differentiable and $|f^{(n)}| \leq 1$ for every n. Show that if f has infinitely many zeros in a bounded set it must be the constant zero function.