**MAT 536, Spring 2024, Sample Final Exam**

| Name | ID |
|---|---|

| Part I | Part II | Total |
|---|---|---|

Part I: True/False. Put a "T" or "F" in each box. 2 points each, 20 points total.

(1) ☐ $(1+i)^2 = 2i$

(2) ☐ $\sum_{n=1}^{\infty} \frac{(\pi i)^n}{n!} = -2$

(3) ☐ If $f$ is bounded and analytic on $\mathbb{D} = \{|z| < 1\}$ its Taylor series converges uniformly to $f$ on $\mathbb{D}$.

(4) ☐ If $f$ is analytic and non-zero on a simply connected domain $\Omega$ there exists an analytic $g$ on $\Omega$ so that $e^g = f$.

(5) ☐ The conformal map of the unit disk to a bounded polygonal domain must extend continuously to the boundary.

(6) ☐ If $\{f_n\}$ are analytic functions on a domain $\Omega$ that converge pointwise on $\Omega$ to a function $f$, then $f$ is analytic.

(7) ☐ If $\mathcal{F}$ is the family of analytic functions $f$ on $\mathbb{D}$ so that $|f'|$ is bounded by 1, then $f$ is a normal family.

(8) ☐ If $\mathcal{F}$ is a Perron family, then $-\mathcal{F} = \{-f : f \in \mathcal{F}\}$ is also a Perron family.

(9) ☐ If $u$ is bounded and harmonic on $\mathbb{D}$, then its harmonic conjugate is also bounded on $\mathbb{D}$.

(10) ☐ $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta} = 2\pi/3$.

Part II: Do three of the following four problems. Mark the boxes next to the problems you want graded. 10 points each, 30 points total.

(1) ☐ Prove that if $f = u + iv$ is analytic on the plane and $|u| \leq |v| + 1$, then $f$ is constant.

(2) ☐ Suppose $f$ is entire (analytic on whole plane). Must $f$ have a fixed point (a solution of $f(z) = z$)? Prove or give a counterexample.

(3) ☐ Suppose $f$ and $g$ are entire and $f^3 + g^3 = 1$. Prove $f$ and $g$ are constant. (Hint: apply Picard's theorem to $f/g$).

(4) ☐ Suppose $f$ is analytic on $\mathbb{D} = \{|z| < 1\}$ and that it extends to be continuous and non-vanishing on $\mathbb{T} = \{|z| = 1\}$. Prove there is a analytic $g$ on $\mathbb{D}$ so that $|g| = |f|$ on $\mathbb{T}$ and $g$ is non-vanishing on all of $\overline{\mathbb{D}}$.