MAT 487 Spring 2014, Tutorial on Rudin's *Prin. of Math. Analysis* **, Midterm**

Name	ID

Problem 1 (10 points): Give the correct definition or statement.

(1) Define pointwise convergence of a sequence of functions $\{f_n\}$.

(2) Define uniform convergence of a sequence of functions $\{f_n\}$.

(3) State the Cauchy criterion for uniform convergence of a sequence of functions.

(4) Define the supremum norm.

(5) Define equicontinuous family.

(6) Define uniformly closed algebra of functions.

(7) State the Stone-Weierstrass theorem for real valued functions.

(8) Define analytic function.

(9) State the fundamental theorem of algebra.

(10) Define orthogonal system of functions on $[a, b]$.

Problem 2 (10 points): Give an example of each, or explain why it can't exist:

(1) A sequence of functions on $[0, 1]$ that converges pointwise, but not uniformly.

(2) A sequence of functions on $[0, 1]$ that converges uniformly, but not pointwise.

(3) A sequence $\{f_n\}$ on $[0, 1]$ that converges pointwise to 0 everywhere, but $\int_0^1 f_n dx \not\to 0$.

(4) A sequence of functions $\{f_n\}$ on $[0, 1]$ so that $\int_0^1 f_n dx \to 0$, but f_n does not converge pointwise at any point.

(5) A continuous function on the plane that is not analytic.

(6) A power series with radius of convergence 1 that converges everywhere on the unit circle.

(7) A power series that converges in $\{|z| < 1\}$, but diverges everywhere on $\{|z| = 1\}$.

(8) An analytic function on the whole plane with no zeros.

(9) A periodic function f on $\mathbb{R}$ whose Fourier series converges uniformly to f.

(10) A linear and quadratic function that are orthogonal on $[0, 1]$.

Problem 3 (10 points): Give a complete and correct proof of two of the following statements (your choice). You may use results from the text if they are correctly quoted.

(1) If f is continuous on $[0, 1]$, show that f can be uniformly approximated by a polynomial that only has even powers of x.

(2) Prove that $f(x) = \sum_{n=1}^{\infty} x^n e^{-n|x|}$ is a continuous function.

(3) Define $f(x) = \exp(-1/x^2)$ for $n \neq 0$ and $f(0) = 0$. Show that the Taylor series for f at 0 converges everywhere, but does not converge to f, except at 0.

(4) Suppose $\{S_n\}$ are the partial sums of the Fourier series of a continuous, 2π-periodic function f on $\mathbb{R}$. Prove that $\int_0^{2\pi} |f(x) - S_n(x)|dx \to 0$, as $n \to \infty$.