MAT 487 Spring 2014, Tutorial on Rudin's *Prin. of Math. Analysis* **, Final**

Name	ID

Problem 1 (10 points): Give the correct definition or statement.

(1) Define differentiability of a function $f : \mathbb{R}^n \to \mathbb{R}^n$.

(2) State the contraction mapping principle.

(3) State the inverse function theorem.

(4) State the rank theorem.

(5) Define a partition of unity.

(6) Define a differential form of order k on $\mathbb{R}^n$.

(7) State Stokes theorem.

(8) Define measurable set.

(9) State the monotone convergence theorem.

(10) State the Lebesgue dominated convergence theorem.

Problem 2 (10 points): Give an example of each, or explain why it can't exist:

(1) A sequence of functions on $[0, 1]$ that converges pointwise, but not uniformly.

(2) A sequence $\{f_n\}$ on $[0, 1]$ so that $f_n(x) \to 0$ for every x but $\int_0^1 f_n dx \not\to 0$.

(3) A function $f(x, y)$ so that the x and y partials exist, but f is not differentiable at $(0, 0)$.

(4) A subset $E \subset \mathbb{R}^2$ and a strict contraction $f : E \to E$ that has no fixed point.

(5) A function that is C^1 on $\mathbb{R}$, but is not C^2.

(6) A measurable function on $\mathbb{R}$ that is nowhere continuous.

(7) A sequence of functions on $[0, 1]$ so that $\int_0^1 |f_n| dx \to 0$ but $f_n(x)$ does not converge to zero for any x.

(8) An integrable function f on $[0, 1]$ so that f^2 is not integrable.

(9) An example of strict inequality in Fatou's theorem.

(10) An uncountable set of Lebesgue measure zero.

Problem 3 (10 points): Give a complete and correct proof of two of the following statements (your choice). You may use results from the text if they are correctly quoted.

(1) If $f \geq 0$ on E and $\int_E f dx = 0$, then $f = 0$ almost everywhere on E.

(2) If f_n is a sequence of measurable functions, then the set where f_n converges is measurable.

(3) Prove that there is a non-measurable set in $[0, 1]$.

(4) Prove that the continuous functions are dense in the integrable functions (L^1 norm).