MAT 532, Stony Brook University, Fall 2017

Problem Set on Chapter 0 (prerequisites)

Due, Thursday, Sept 7

(1) Construct a closed, uncountable subset of $[0,1]$ that contains no rational numbers.

(2) Construct a continuous real-valued function on $\mathbb{R}$ that is differentiable at every irrational number, but not differentiable at any rational number.

(3) Construct a continuous function $f : [0,1] \to [0,1]$ that takes every value uncountable often.

(4) Let $\mathbf{C} \subset [0,1]$ be the middle thirds Cantor set. Which translates of $\mathbf{C}$ intersect itself? More precisely, identity the set $\{t : \mathbf{C} \cap (\mathbf{C} + t) \neq \emptyset\}$, where $E + t = \{x + t : x \in E\}$.

(5) Give an example of a subset $E \subset [0,1]$ that is a countable union of closed sets, but is not a countable intersection of open sets.