MAT 401, fall 2009

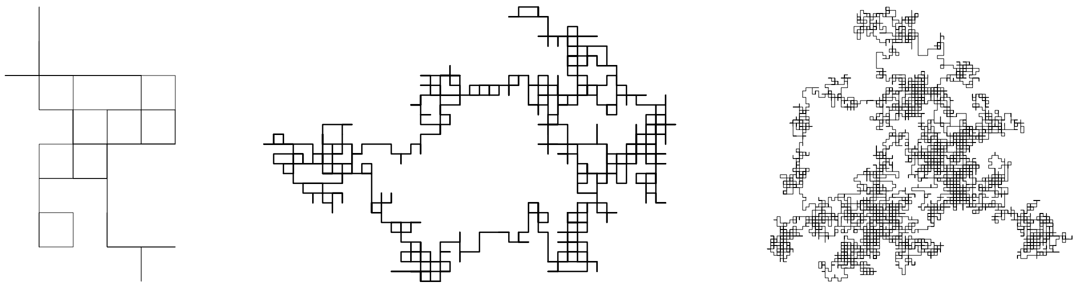


FIGURE 1. Random walks on a square grid with 100, 1000 and 10,000 steps.

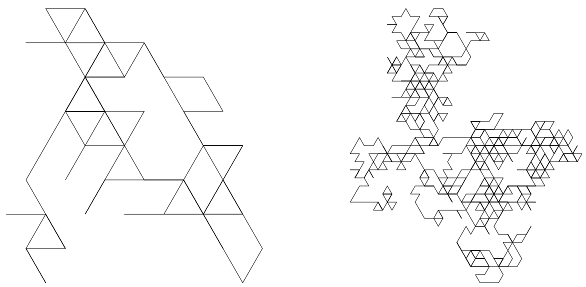


FIGURE 2. Random walks on a triangular grid with 100, 1000 and 10,000 steps.

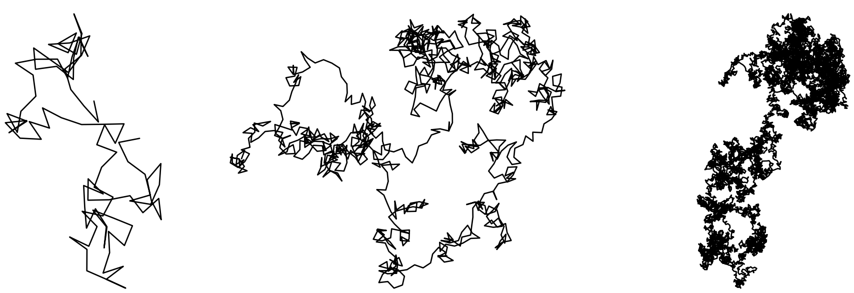


FIGURE 3. Random paths formed by stepping unit distance in a randomly chosen direction. The pictures show paths with 100, 1000 and 10,000 steps. Note that regardless of the discrete random walk, at large scales the results all look the same.

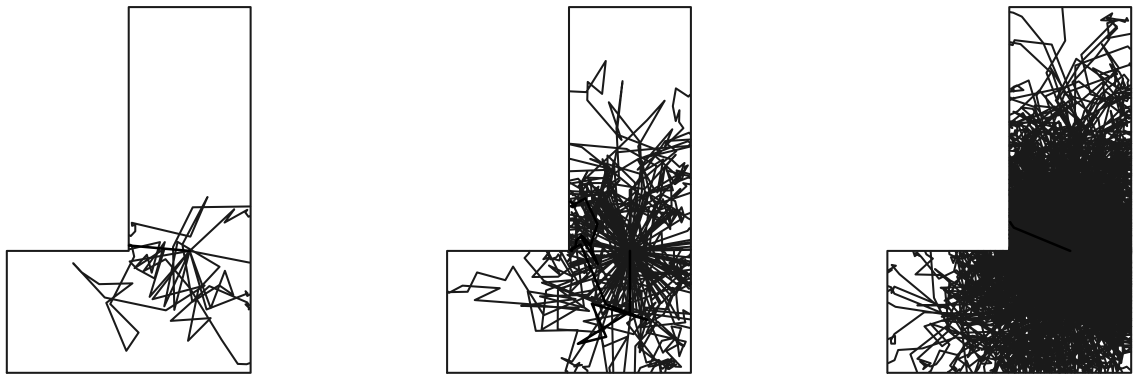


FIGURE 4. These show the polygon with 10, 100, and 1000 random walks. Note how hard it is for the remote edges to get hit. After 1000 attempts the top edge still has not been hit. If the smallest harmonic measure of any edge is ϵ the we expect to need $1/\epsilon$ to get even one hit on that edge.

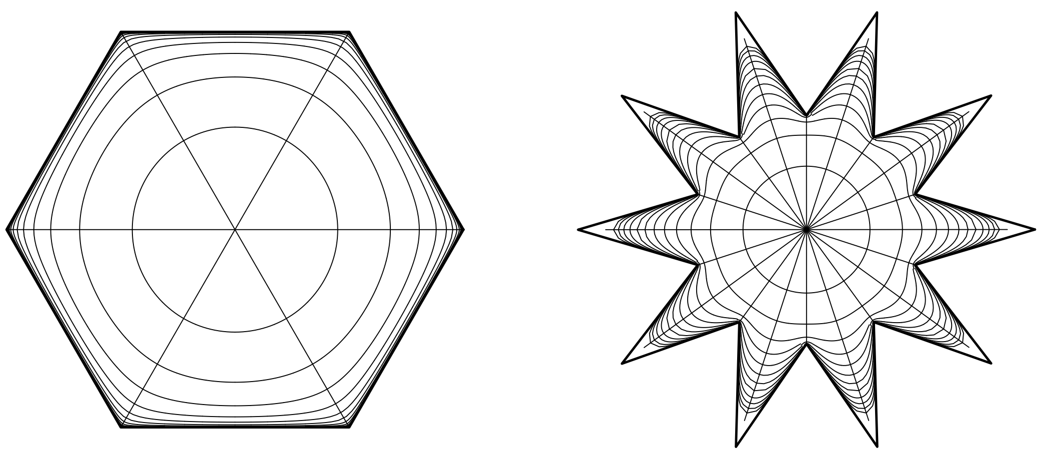


FIGURE 5. Polygons for which every edge has the same harmonic measure.

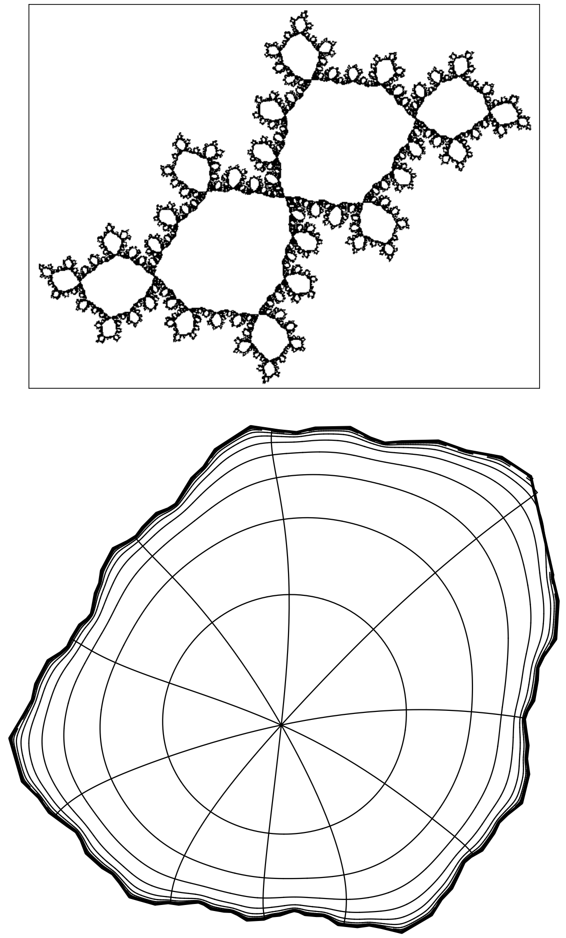


FIGURE 6. An approximation of the Siegel disk for the polynomial $g(z) = z^2 + \lambda z, \qquad \lambda = \exp(i2\pi 4^{-1/3})$. The top figure shows the whole Julia set (this picture is courtesy of Jack Milnor and appears in Section 11 of his book on complex dynamics). The lower pictures shows the level lines and 10 equally spaced radial lines for the map of the disk onto the fixed component of the Fatou set (the large component in the lower left).

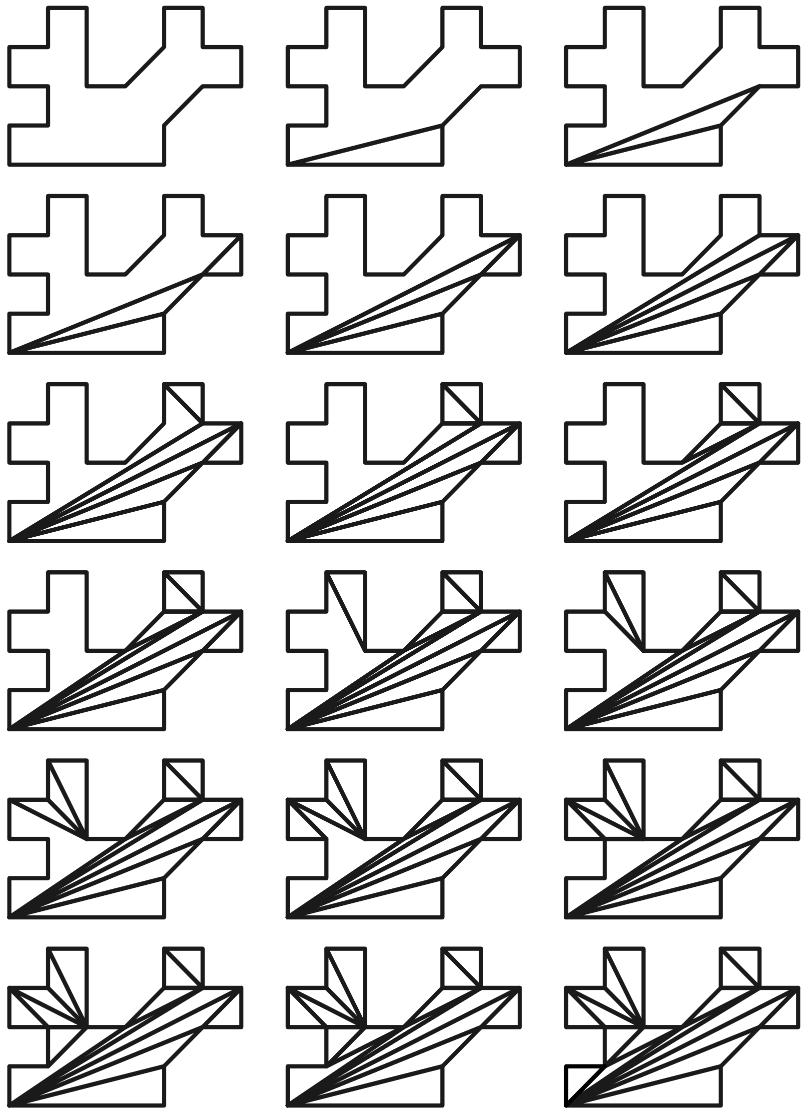


Figure 7.

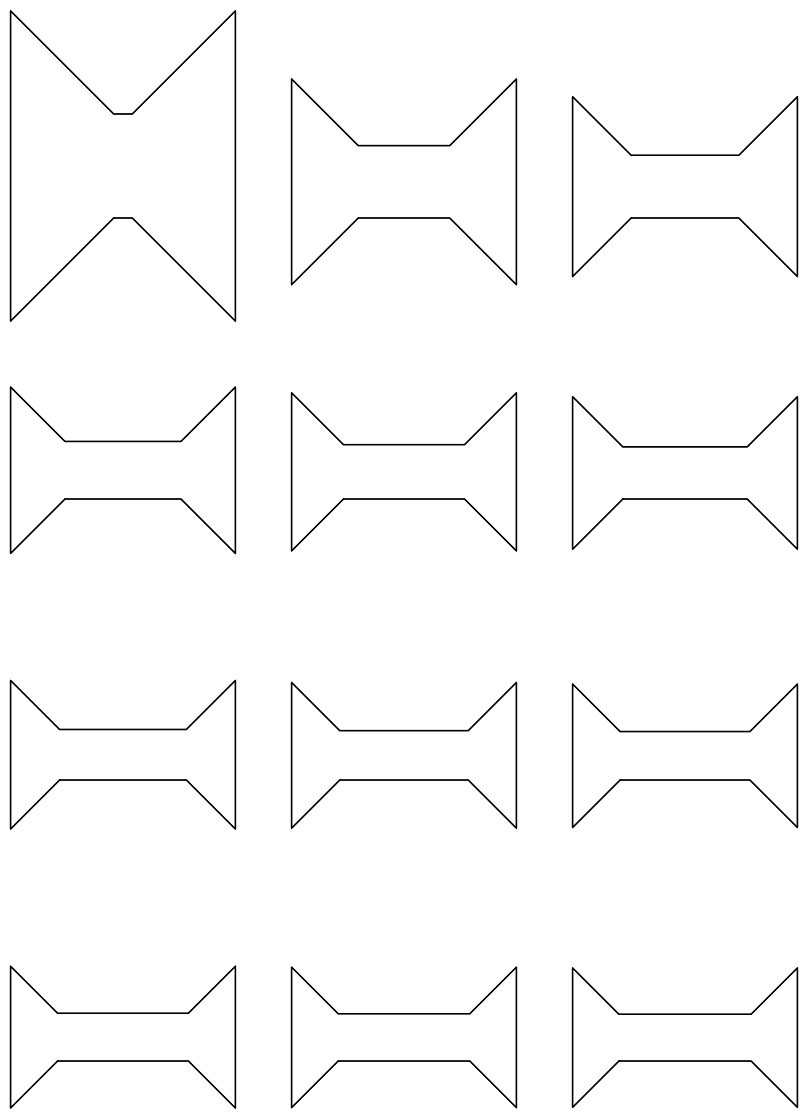


Figure 8.

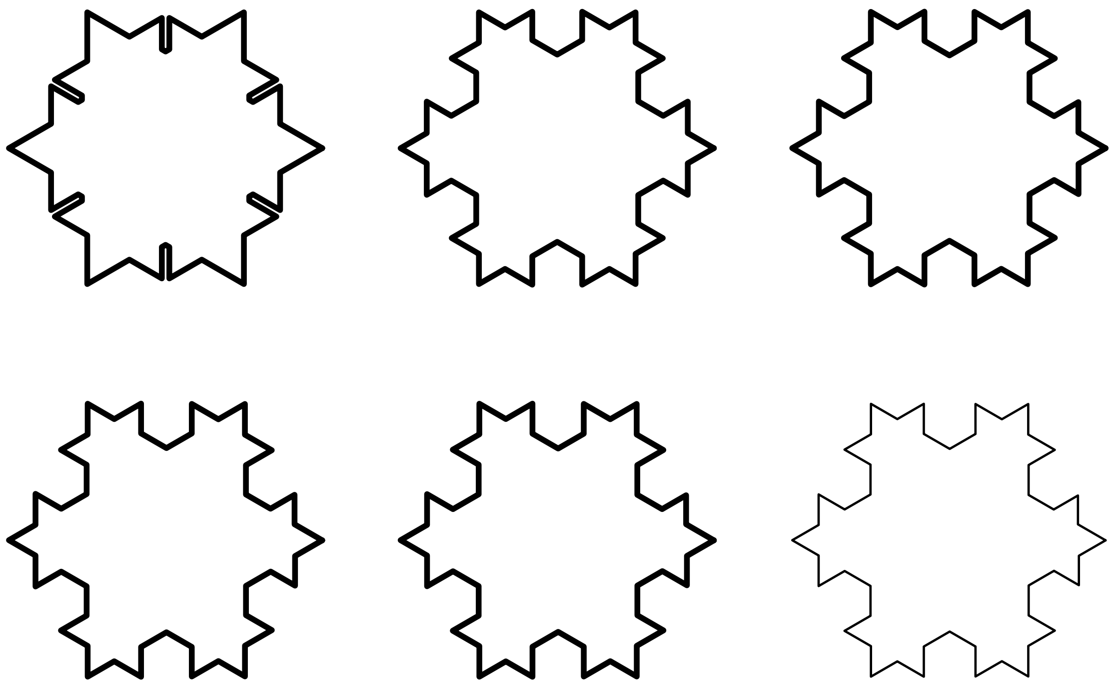


FIGURE 9. Here is Davis' method applied to the second generation of the von Koch snowflake. The first picture shows the polygon generated when all the sides are assumed to have equal harmonic measure. Even after just one iteration, the figure is much improved and virtually indistinguishable from the target polygon after a few more iterations. The QC distances to the true polygon are

$$13.6017, 1.14795, 1.14117, 1.13452, 1.12169, 1.0999.$$

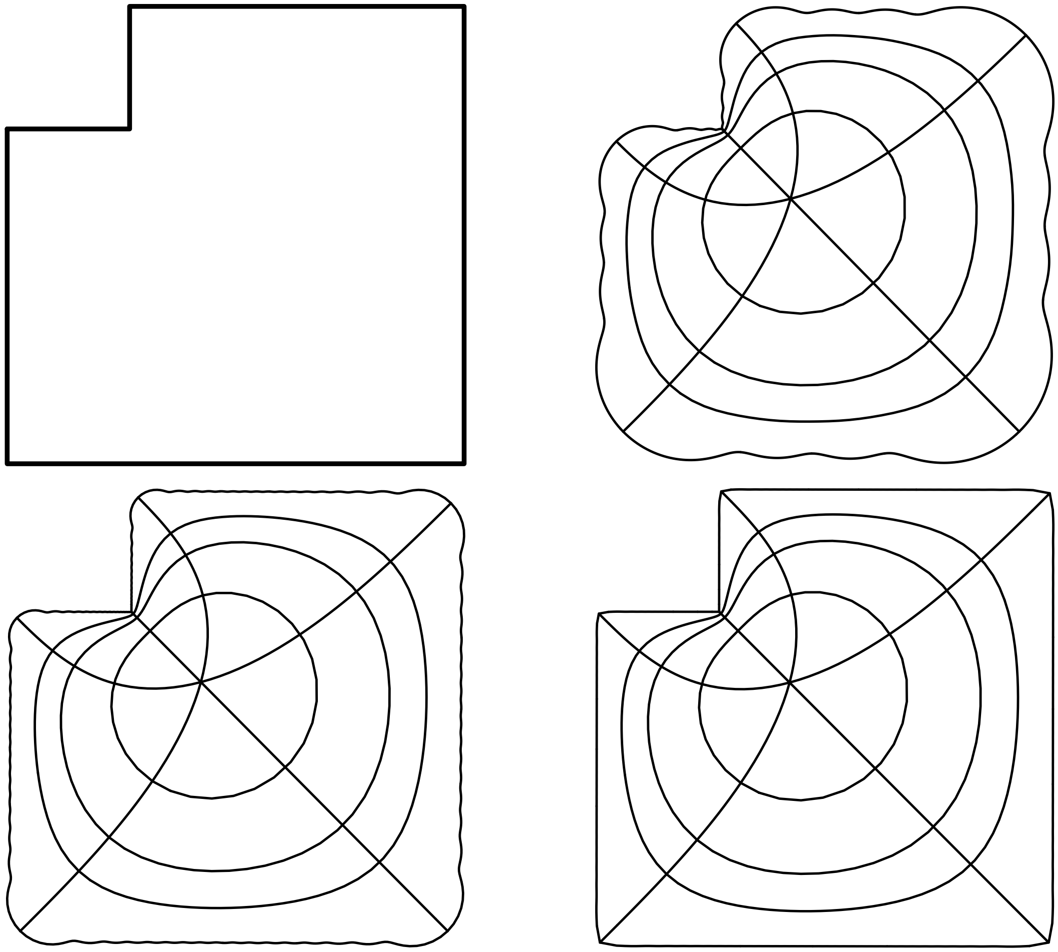


Figure 10. In the upper left is the target "L"-shaped polygon. The parameters for Schwarz-Christoffel are taken to be equidistributed in this example. The next three figures show the images of the disk under the power series for the map with truncations at order $20, 100, 1000$.

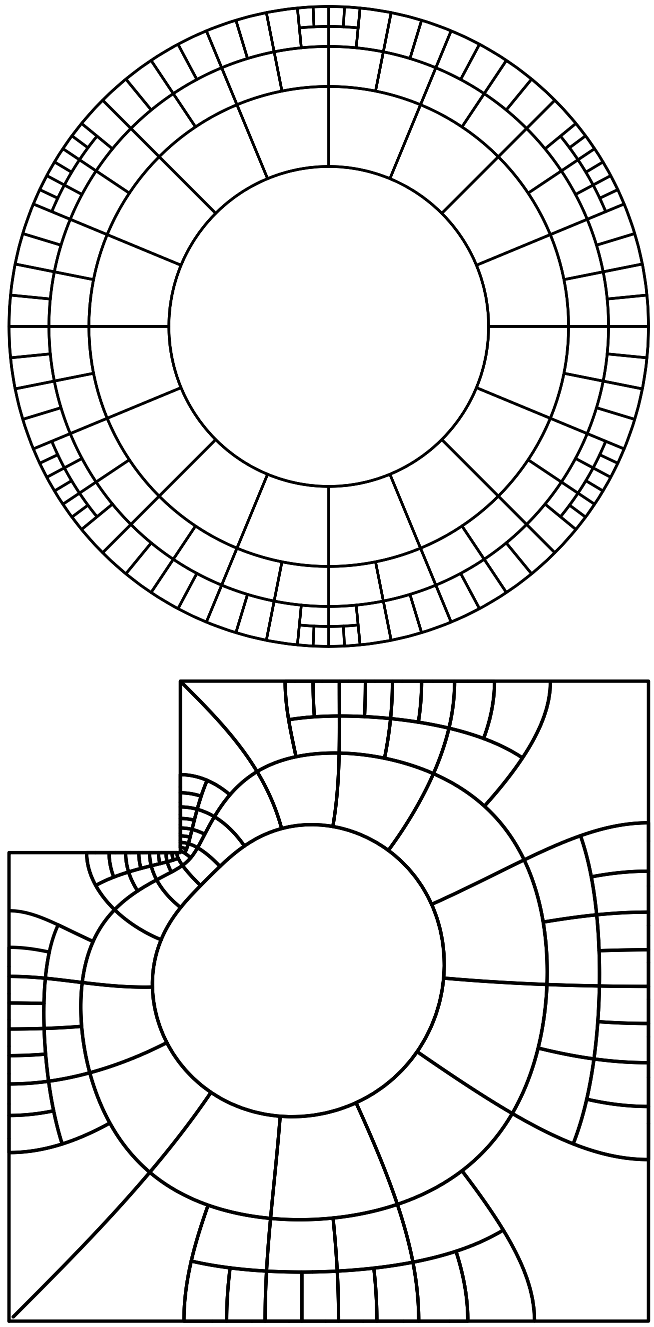


Figure 11.

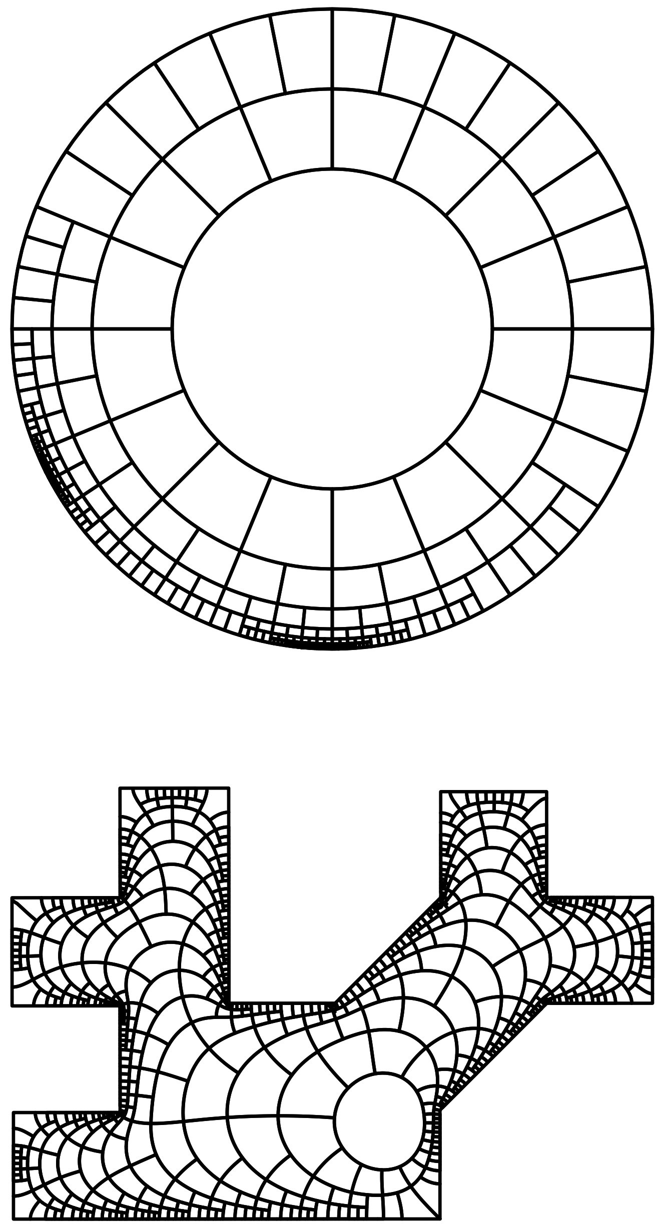


FIGURE 12.

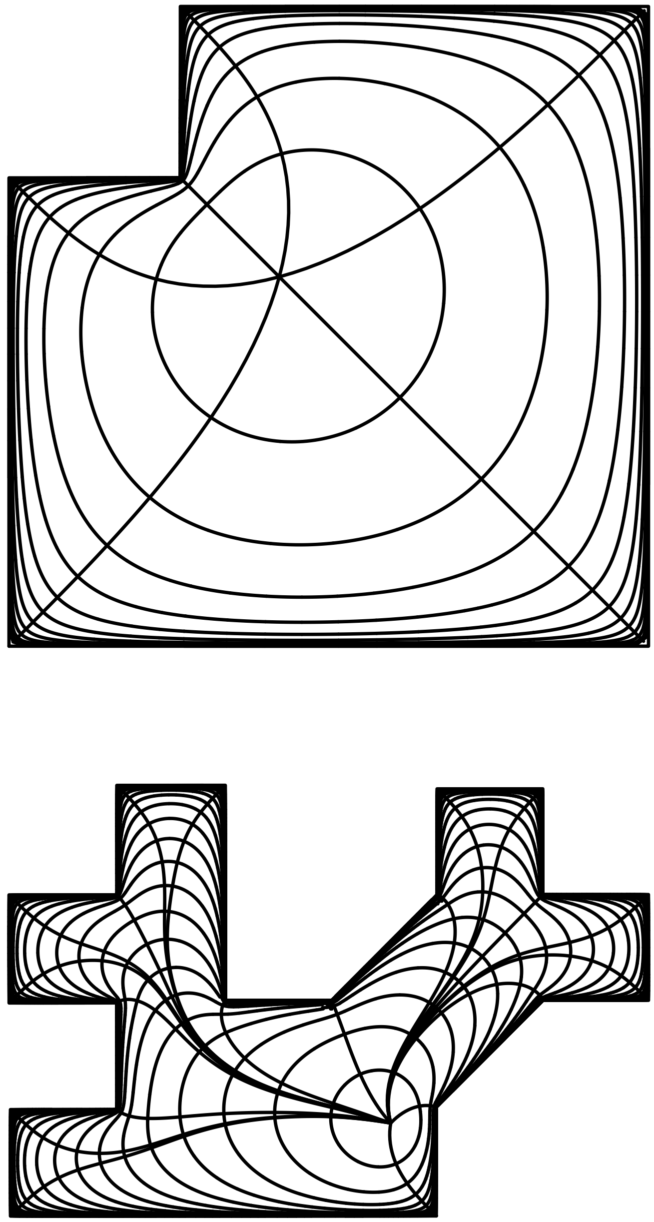


FIGURE 13. For the polygons in Figure 11 and **??** we have used the multiple power series representation to plot the images of the circles of radius $1 - 2^{-n}$ and the radial segments that end at the vertices. As before, polynomials of degree 10 are used for all approximations.

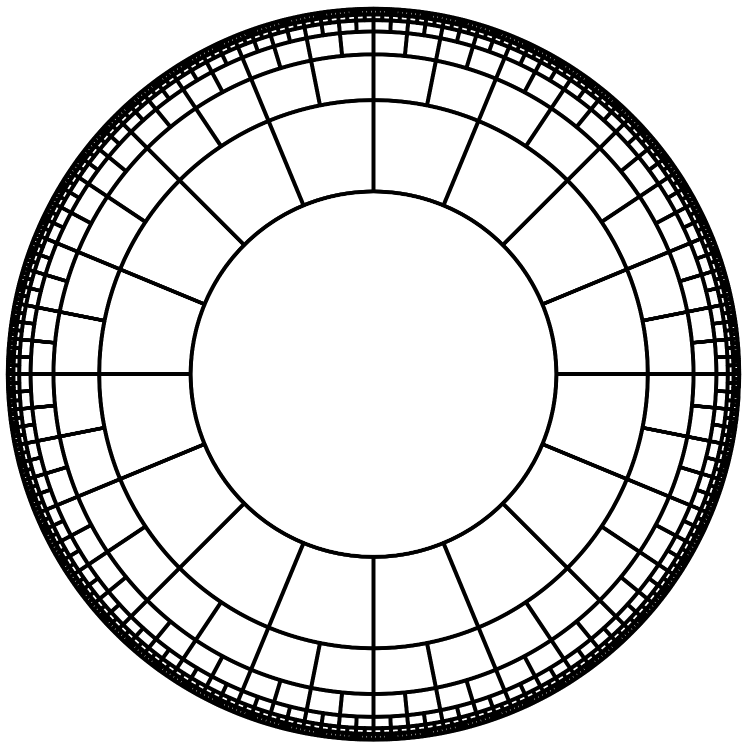


FIGURE 14. A decomposition of the disk into Whitney squares. There is a central disk which has 16 children, each of which has 2 children, and so on towards the boundary. The choice of 16 and 2 is arbitrary, but made here so that the Whitney boxes are "roundish" and so that each box is contained in a disk whose double is still in the unit disk. This means that any holomorphic function on the unit disk has a power series expansion around the center of each Whitney box which converges geometrically fast on the box.

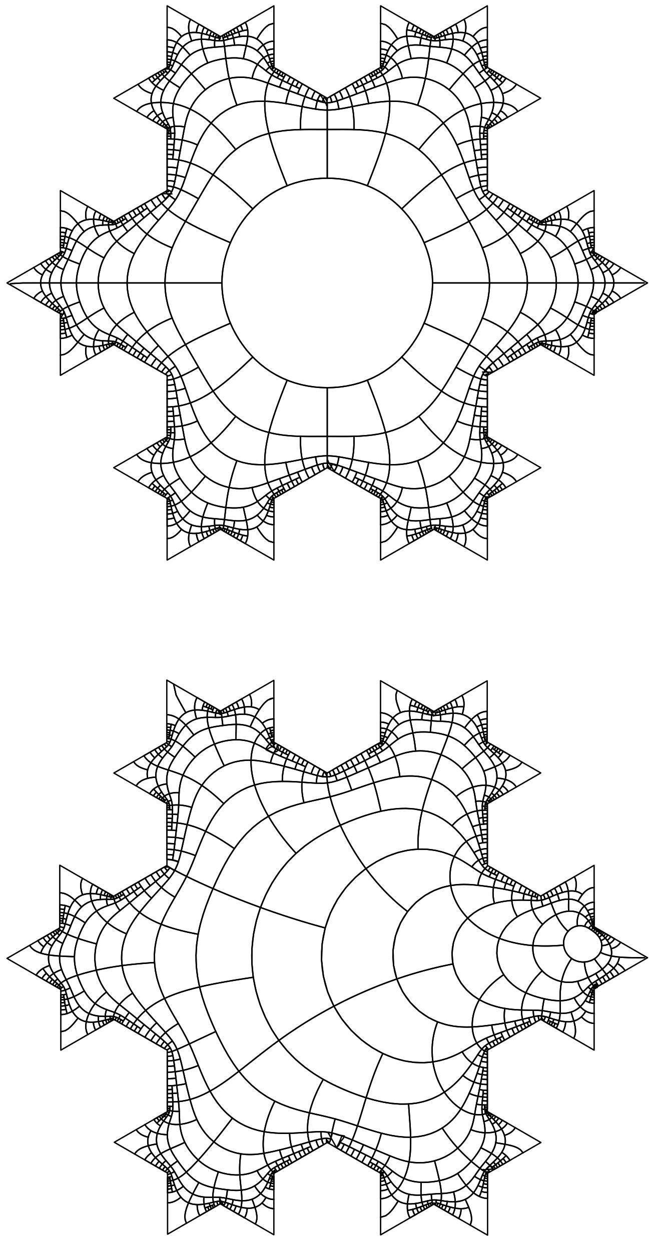


FIGURE 15. Images of the Whitney boxes for the second generation von Koch snowflake.

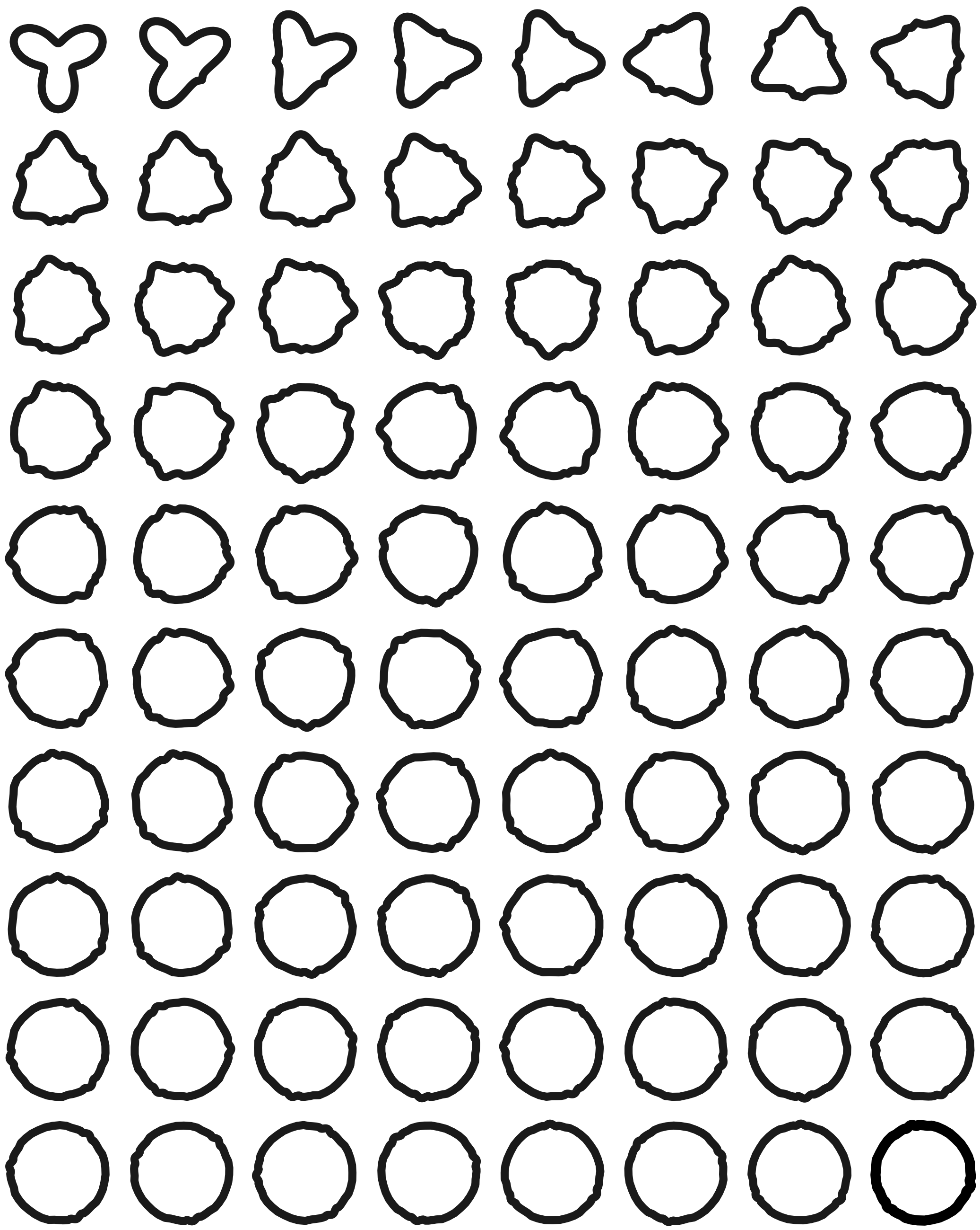


Figure 16. This shows the first 80 iterations of Koebe's method for the same domain as in Figure ??

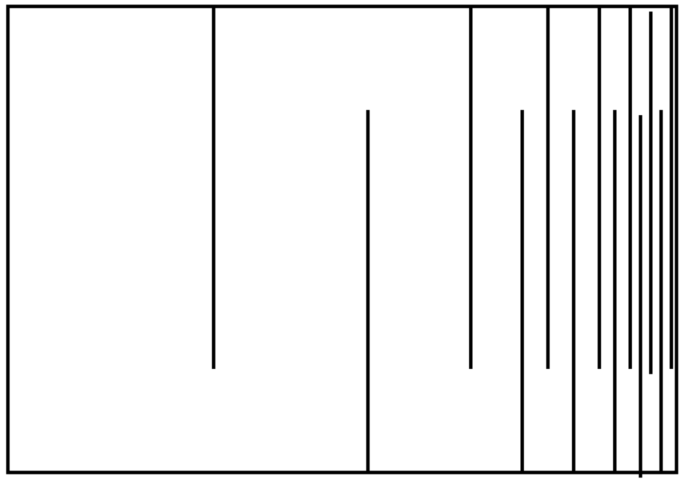


FIGURE 17. An example of a domain with a non-locally connected boundary. The Riemann map onto the interior of this domain fails to have a continuous boundary extension at one point. Examples can be constructed where it fails to have a continuous boundary extension at any point.

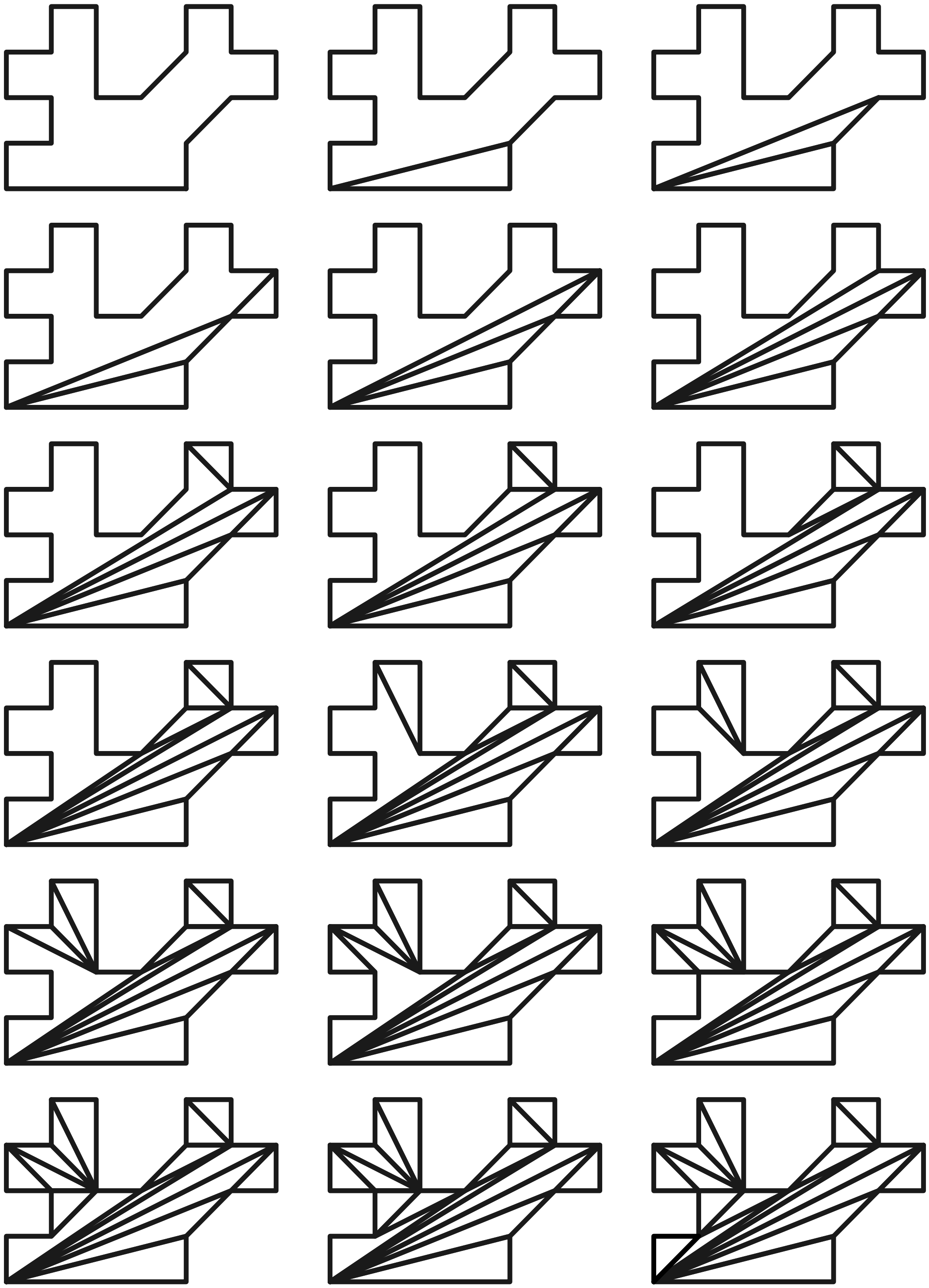


Figure 18. Another triangulation of a simple polygon.

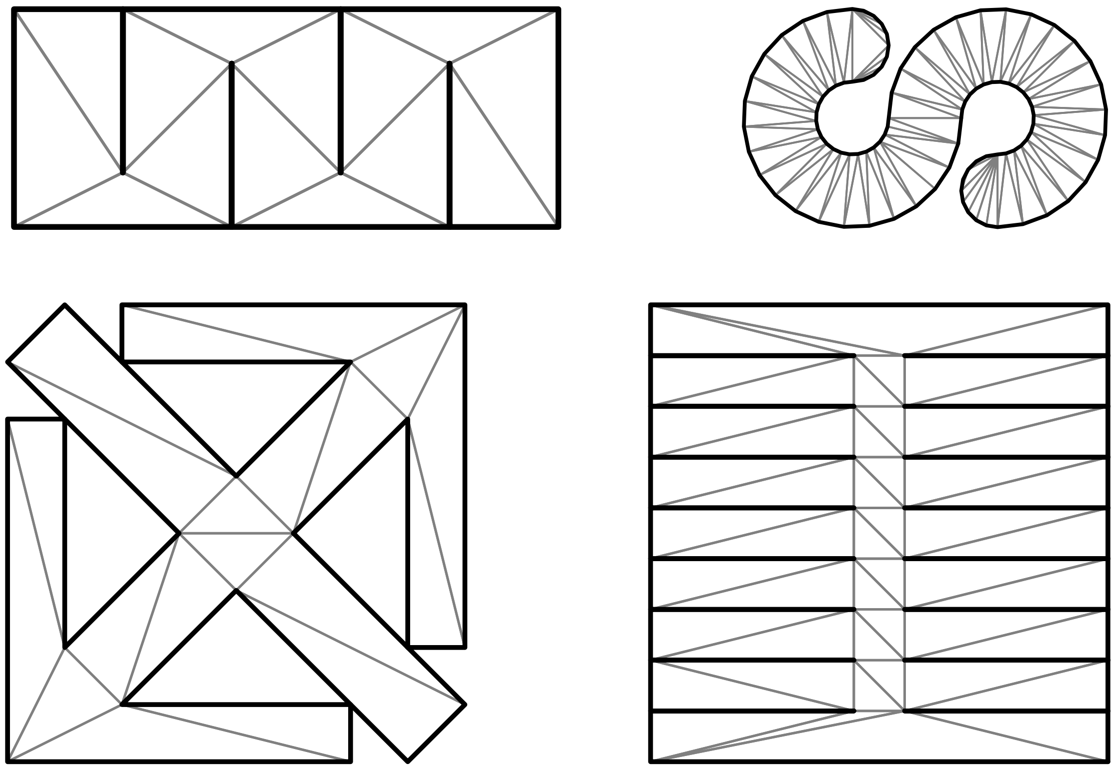


FIGURE 19. Delaunay triangulations of some more polygons.

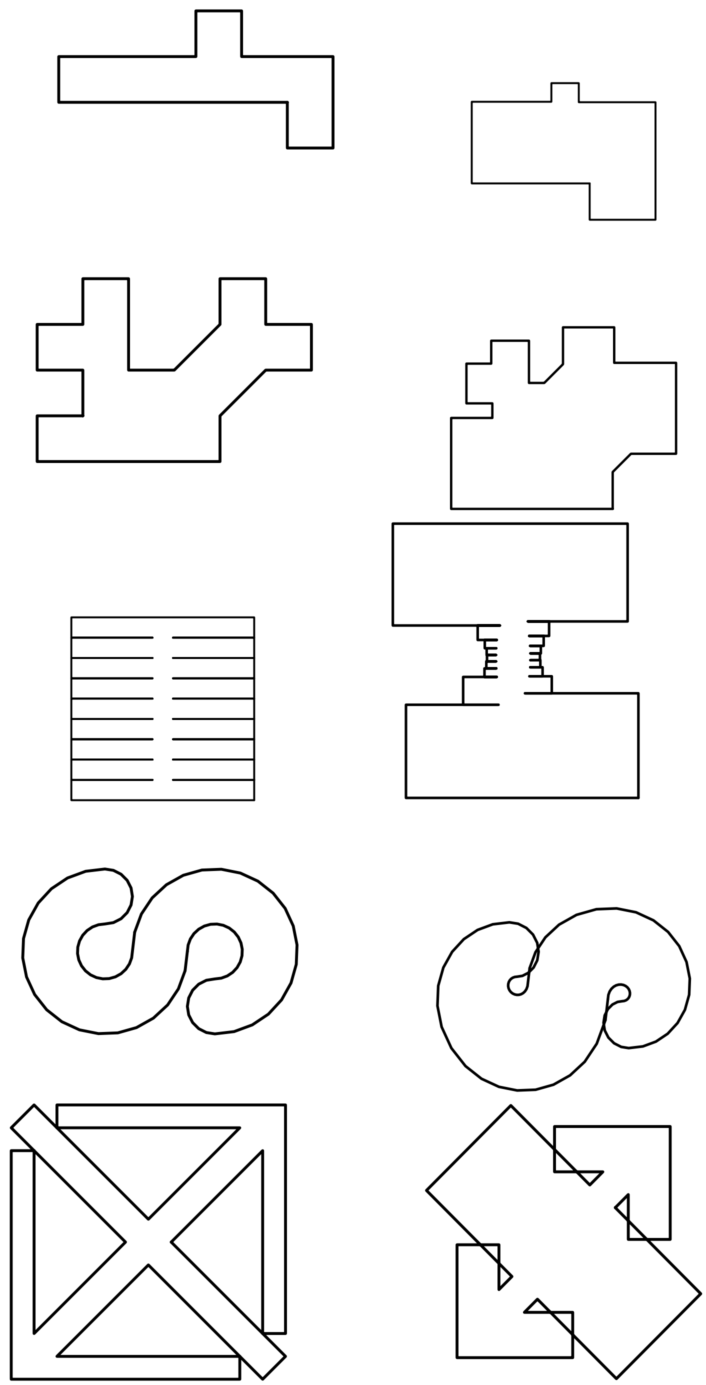


FIGURE 20. On the left are various polygons and on the right is the corresponding Schwarz-Christoffel image using the parameters given by the CRDT intial guess. We have not performed the first step of CRDT by adding extra vertices. Note that in some cases the SC image is not simple.

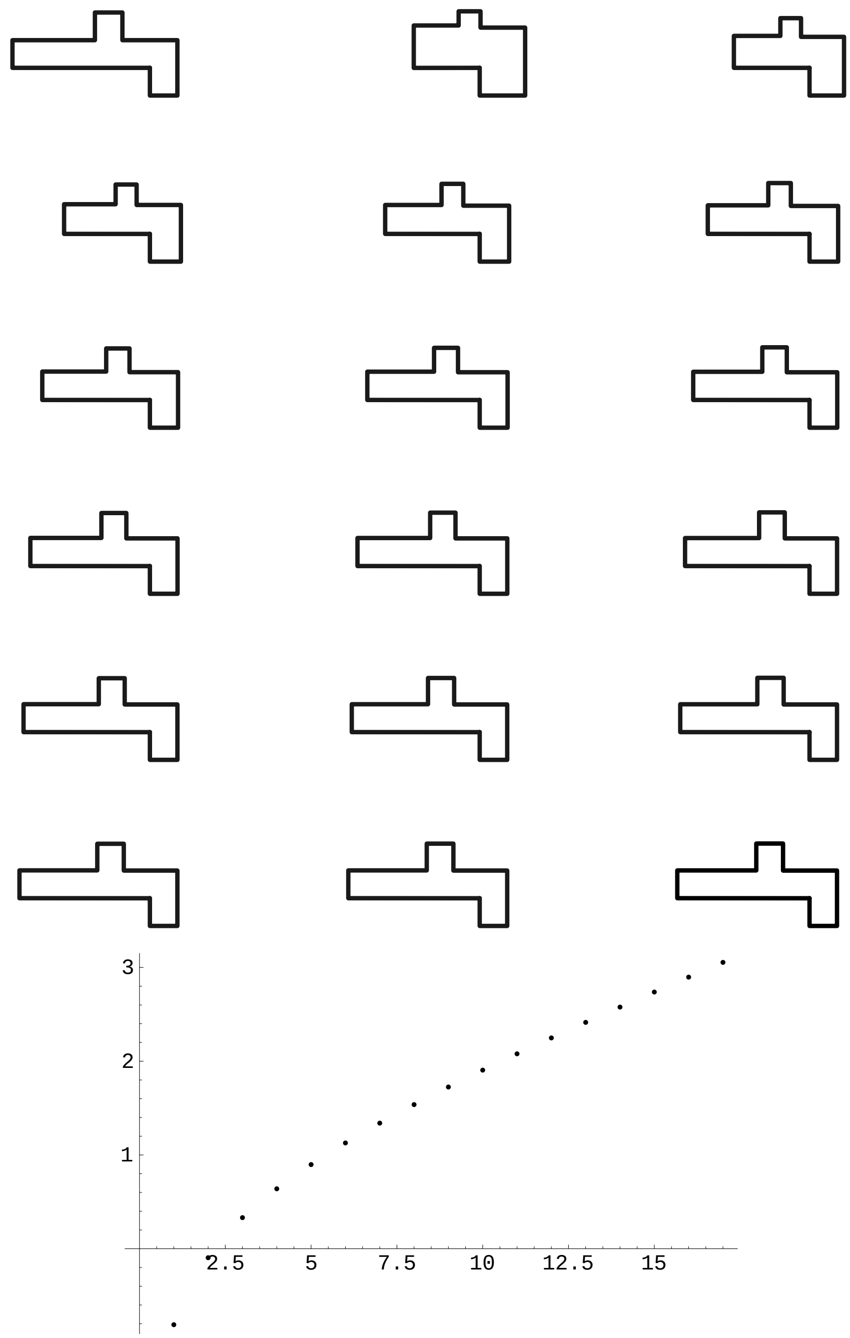


Figure 21. Simple CRDT.

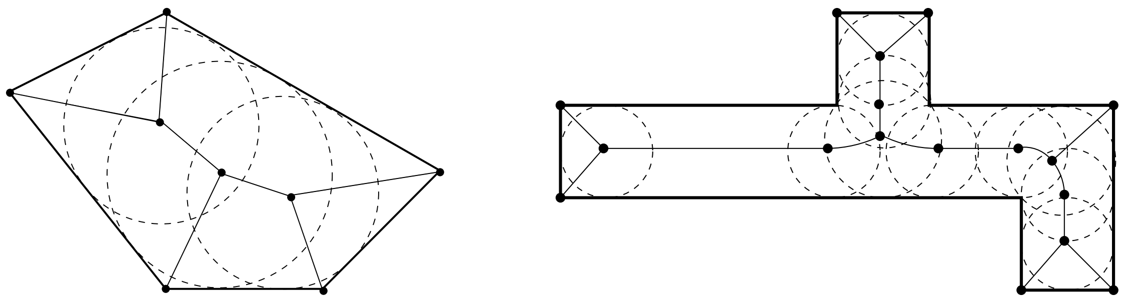


FIGURE 22. Examples of medial axes of polygons

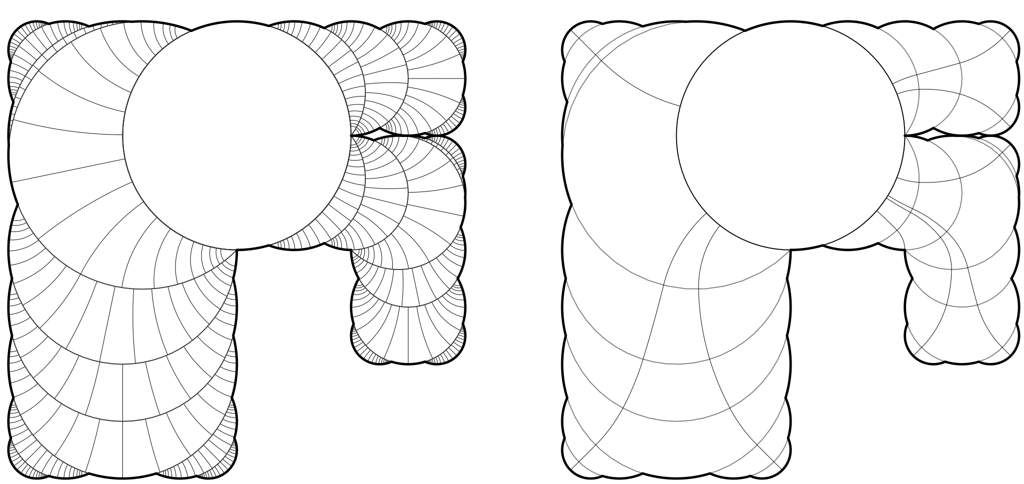


FIGURE 23. A finite union of disks written a union of a root disk D_0 and several crescents. Each crescent is foliated by circular arcs orthogonal to the boundary and following the foliation gives a map from $\partial\Omega$ to ∂D_0.

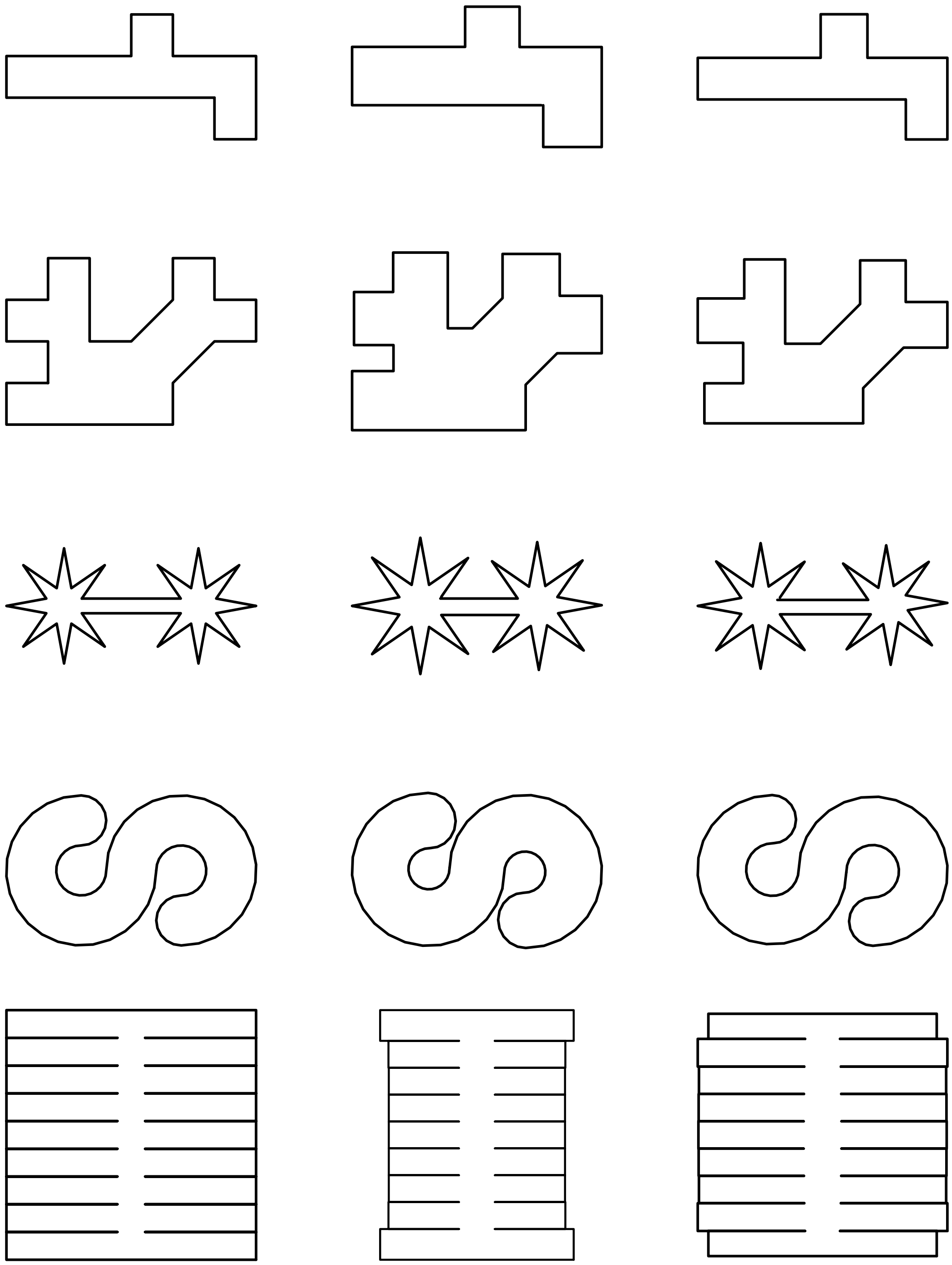
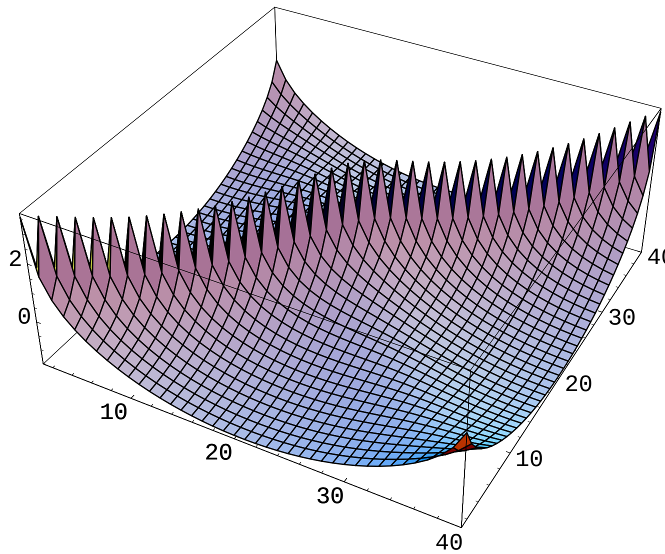


Figure 24.



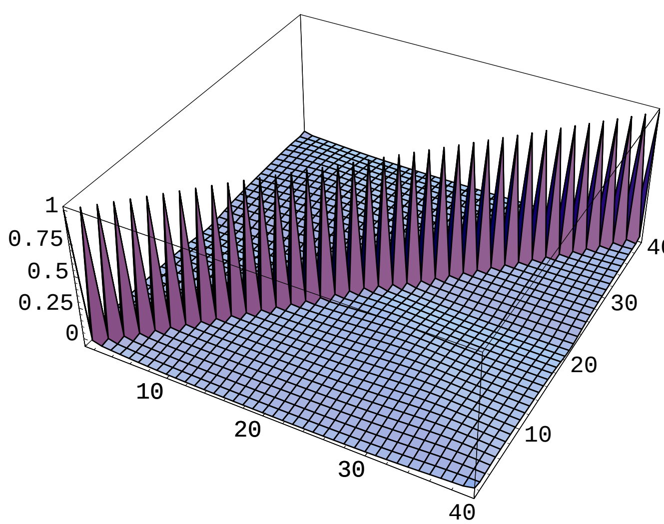


FIGURE 25. These show larger plots of the matrices that arise for the ellipse discretized with 40 points using single logarithmic poles versus the modified subtended angle functions. The second plot should look more like an identity matrix (and it does).