MAT 342 Fall 2016, Sample Midterm 2,
Actual Midterm is 10:00-10:53am, Wed., November 16, 2016

Name	ID	

THIS EXAM IS WORTH 50 POINTS. EACH QUESTION IS WORTH ONE POINT. NO BOOKS, NOTES OR CALCULATORS ARE ALLOWED.

1-10: Write C (for converges) or D (for diverges) for each sequence or series.

(1) ☐ $(\frac{i}{2})^n$, $n = 1, 2, \ldots$.

(2) ☐ $1 + n$, $n = 1, 2, \ldots$.

(3) ☐ $\sum_{n=0}^{\infty} \frac{1}{n^2}$

(4) ☐ $\sum_{n=0}^{\infty} (\frac{i}{3})^n$

(5) ☐ $\sum_{n=0}^{\infty} \frac{i^n}{n^3}$.

(6) ☐ $i^n + (-i)^n$, $n = 1, 2, \ldots$.

(7) ☐ $|\exp(i\pi n/4)|$, $n = 1, 2, \ldots$.

(8) ☐ $\sum_{n=-\infty}^{\infty} \frac{i^n}{1+n^2}$

(9) ☐ $\sum_{n=-\infty}^{\infty} 2^{-n}$

(10) ☐ $\frac{n^2+n-1}{2n^2-3}$, $n = 1, 2, \ldots$.

11-20: Match each function with its Maclaurin series.

(11) ☐ $z^2 \cos(z)$

(12) ☐ $\sinh(z)$

(13) ☐ $\frac{1}{(1-z)^2}$

(14) ☐ $\log(1-z)$

(15) ☐ $\cosh(2z)$

(16) ☐ e^{-z}

(17) ☐ $\cos(z)$

(18) ☐ $\frac{1}{1-z}$

(19) ☐ $\sin(z)$

(20) ☐ $\exp(z^2)$

A. $z^3 - \frac{1}{6}z^5 + \frac{1}{120}z^7 - \ldots$

B. $1 - \frac{1}{2}z^2 + \frac{1}{24}z^4 - \ldots$

C. $z^3 - \frac{1}{2}z^5 + \frac{1}{24}z^7 - \ldots$

D. $1+2z+3z^2+4z^3+5z^4+\ldots$

E. $1 + z + z^2 + z^3 + \ldots$

F. $1 + z^4 + z^6 + z^8 + \ldots$

G. $1 + z + \frac{1}{2}z^2 + \frac{1}{6}z^3 + \ldots$

H. $z + \frac{1}{2}z^2 + \frac{1}{3}z^3 + \frac{1}{4}z^4 + \ldots$

I. $z + z^2 + z^3 + z^4 + \ldots$

J. $1 - \frac{1}{2}z^4 + \frac{1}{24}z^6 - \ldots$

K. $1 - z + \frac{1}{2}z^2 - \frac{1}{6}z^3 + \ldots$

L. $1 + 2z^3 + \frac{2}{3}z^4 + \ldots$.

M. $1 + z^2 + \frac{1}{2}z^4 + \frac{1}{6}z^6 + \ldots$

N. $z + \frac{1}{6}z^3 + + \frac{1}{120}z^5 + \ldots$

P. $1 + \frac{1}{2}z^2 + \frac{1}{24}z^4 + \ldots$

Q. none of the above

21-30: Write T (for true) or F (for false) in each box.

(21) $\square$ If $\sum_{n=0}^{\infty} a_n z^n$ converges at $z = 1$, then if converges for all $|z| \leq 1$.

(22) $\square$ If $\sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} b_n z^n$ for all $|z| < 1$, then $a_n = b_n$ for all $n = 0, 1, 2, \ldots$.

(23) $\square$ If $\sum_{n=0}^{\infty} a_n z^n$ converges at $z = 1$, then the $\{a_n\}$ are a bounded sequence.

(24) $\square$ The power series for $\frac{1}{z^2+1}$ at $z = 2$ has radius of convergence equal to 1.

(25) $\square$ If $\sum_{n=0}^{\infty} a_n z^n$ converges at $z = 1$, then if converges for all z with $|z| = 1$.

(26) $\square$ If f has a power series expansion on $|z - 1| < 2$, then f is analytic in that disk.

(27) $\square$ The function $f(z) = \sin(x)$ has a convergent Laurent series expansion on $|z| > \pi/2$.

(28) $\square$ If f has an essential singularity at 0 it takes every complex value in every neighborhood of 0.

(29) $\square$ If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$ are both analytic on $|z| < 1$, then $f(z)/g(z) = \sum_{n=0}^{\infty} \frac{a_n}{b_n} z^n$ for all $|z| < 1$.

(30) $\square$ If $f(z)$ has a pole of order m at $z = 0$, then $g(z) = z^m f(z)$ has removable singularity at 0.

31-35: Compute the residue of each function at the given point.

(31) $\boxed{}$ $\frac{\cos z}{z^4}$ at $z = 0$.

(32) $\boxed{}$ $\exp(z + z^2)$ at $z = 0$.

(33) $\boxed{}$ $\frac{1}{1+z^6}$ at $z = i$.

(34) $\boxed{}$ $\frac{\mathrm{Log} z}{z^2+1}$ at $z = i$.

(35) $\boxed{}$ $\frac{4z-5}{z(z-1)}$ at $z = \infty$.

36-40: For each function and point, identity the type of singularity: R = removable, P = pole, E = essential singularity.

(36) $\boxed{}$ $f(z) = \sin(\frac{1}{1-z})$, $z = 1$.

(37) $\boxed{}$ $f(z) = \exp(1 + z + z^2)/\sin(z)$, $z = 0$.

(38) $\boxed{}$ $f(z) = \frac{1-\cos z}{z^2}$, $z = 0$.

(39) $\boxed{}$ $f(z) = \frac{1}{1-\cos z}$, $z = 0$.

(40) $\boxed{}$ $f(z) = \frac{z}{\sin z}$, $z = 0$.

41-46 Evaluate $\int_{-\infty}^{\infty} f(x)dx$ where $f(x) = \frac{1}{x^2+2x+2}$, following the steps below.

(41) State the Cauchy residue theorem

(42) Draw a closed contour C_R that contains the interval $[-R, R]$ and so that the integral over the rest of the contour tends to zero as $R \nearrow \infty$. Label the points $-R$ and R.

(43) $\boxed{}$ List all the singularities of f inside the contour.

(44) $\boxed{}$ Compute the residues of f at the singularities inside the contour.

(45) $\boxed{}$ Compute the integral $\int_{-\infty}^{\infty} f(x)dx$.

41-46 Evaluate $\int_{-\infty}^{\infty} f(x)dx$ where $f(x) = \frac{\sin x}{x^2+4x+5}$, following the steps below.

(46) $\boxed{}$ Give the analytic function $f(z)$ that you will apply the Cauchy residue theorem to evaluate this integral.

(47) Draw a closed contour C_R that contains the interval $[-R, R]$ and so that the integral over the rest of the contour tends to zero as $R \nearrow \infty$. Label the points $-R$ and R.

(48) $\boxed{}$ List all the singularities of $f(z)$ inside the contour.

(49) $\boxed{}$ Compute the residues of f at all the singularities inside the contour.

(50) $\boxed{}$ Compute the integral $\int_0^{\infty} f(x)dx$.