MAT 342 Fall 2016, SAMPLE MIDTERM 1,
Actual midterm is 10:00-10:53am, Friday, October 14, 2016 in P-131 Math

Name	ID	

THIS EXAM IS WORTH 50 POINTS. EACH QUESTION IS WORTH ONE POINT. NO BOOKS, NOTES OR CALCULATORS ARE ALLOWED.

1-10 TRUE/FALSE: Write T (for true) or F (for false) in each box.

(1) $(2-3i)-(4+2i) = -2-5i$

(2) $(2+i)(3+i) = 5+5i$

(3) $(1-i)^3 = -1-i.$

(4) $1/i = i$

(5) $e^{\pi i/4} = \sqrt{2}(1+i)$

(6) $e^{100\pi i} = 1$

(7) $\frac{i}{2-i} = \frac{1+2i}{3}$

(8) $\mathrm{Log}(-1) = \pi$

(9) $\arg(1+i) = \{\frac{\pi}{4} + 2\pi n : n \in \mathbb{Z}\}$

(10) $e^{i} = \cos(1) + i\sin(1).$

11-15 Place the letter of the corresponding point in the box. The same letter might be used more than once.

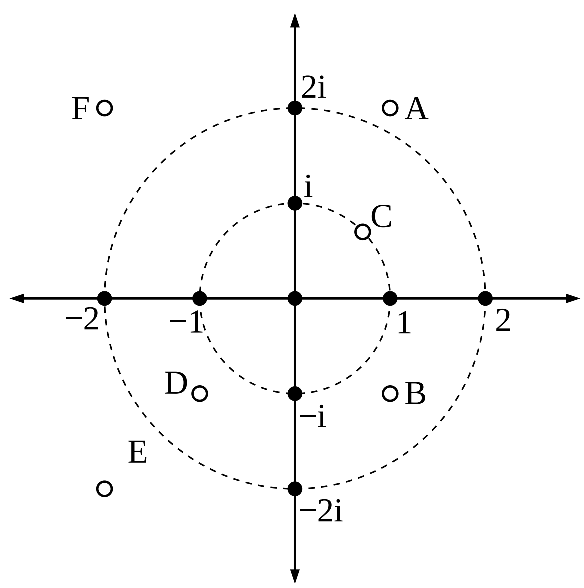
(11) $|z| = \sqrt{5}$

(12) $\mathrm{Re}(z) = -1.$

(13) $z^2 = -2i$

(14) $z = \overline{F}$

(15) $\mathrm{Arg}(z) = -\pi/4.$



16-20 Match each function with its definition. Assume $z = x + iy$.

(16) $\square$ $\sinh(z)$

(17) $\square$ e^z

(18) $\square$ $\sin(z)$

(19) $\square$ $\tan(z)$

(20) $\square$ i^z

A. $\frac{1}{2i}(e^{iz} - e^{-iz})$

B. $\frac{1}{2}(e^{iz} + e^{-iz})$

C. $(-i)\frac{e^{iz}-e^{-iz}}{e^{iz}+e^{-iz}}$

D. $\frac{e^{iz}+e^{-iz}}{e^{iz}-e^{-iz}}$

E. $\frac{1}{2}(e^{z} + e^{-z})$

F. $e^{y}(\cos x + i \sin x)$

G. $e^{x}(\cos x - i \sin x)$

H. $e^x \cos(y)$

I. $e^x \cos(y) + ie^x \sin(y)$

J. $e^{z \log i}$

K. $\frac{1}{2}(e^{z} - e^{-z})$

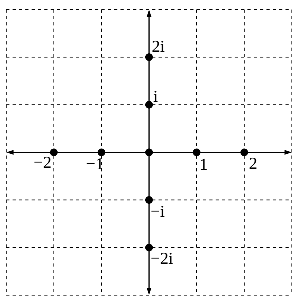
L. $\frac{1}{2} \log \frac{1+z}{1-z}$

M. $e^{i \log z}$

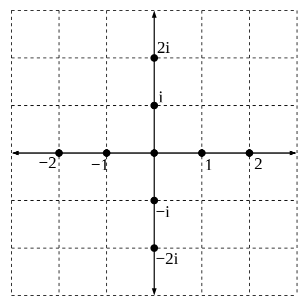
N. none of the above

21-25 Draw the following points or regions as accurately as you can.

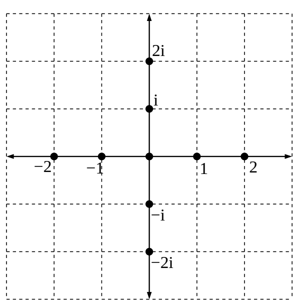
(21) Draw the point $z = 2 - 2i$.



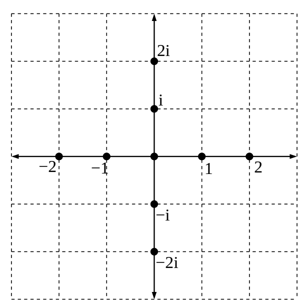
(22) Draw the point $\overline{iz}$, where $z = 1 + i$.



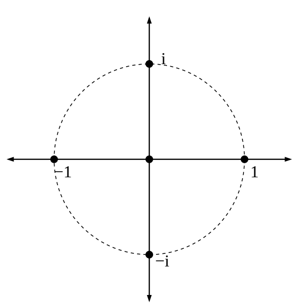
(23) Draw the region $|z + 1 - 2i| \leq 1$.



(24) Draw the region $|\mathrm{Im}(z)| \leq 1$.



(25) Draw all solutions of $z^4 = i$



21-30 TRUE/FALSE: Write T (for true) or F (for false) in each box.

(26) ☐ The function e^z is entire.

(27) ☐ If $f = u + iv$ is analytic and real valued, then f must be constant.

(28) ☐ $|1 - z^2|$ attains a maximum value somewhere on the plane.

(29) ☐ If f has an anti-derivative on domain D, then integral of f around any closed contour in D is zero.

(30) ☐ If f is analytic on a disk D, then f must have an anti-derivative on D.

(31) ☐ The function $\tan(z)$ is analytic on $\{z : |z| < 1\}$.

(32) ☐ A polynomial of degree n must have n distinct zeros.

(33) ☐ $f(x + iy) = 2xy + i(x^2 - y^2)$ is analytic on the plane.

(34) ☐ Suppose $f = u + iv$. If the partials of u and v exist at a point z_0 and satisfy the Cauchy-Riemann equations at z_0, then f is differentiable at z_0 .

(35) ☐ A function u is harmonic if $u_{xx} = u_{yy}$.

36-40: Give a precise statement of each definition or result.

(36) Define "f is analytic in an open set".

(37) State the coincidence principle.

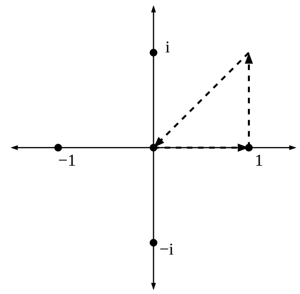
(38) State Cauchy's formula.

(39) State the Cauchy-Riemann equations for $f = u + iv$.

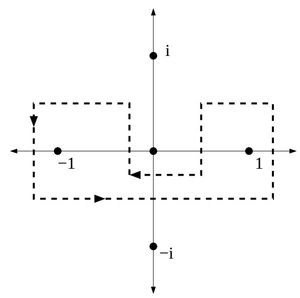
(40) Define simply connected domain.

40-45 Evaluate each integral for the given contour; put your answer in the box.

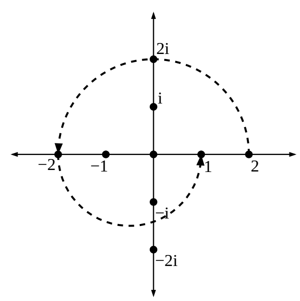
(41) $\int_C \overline{z} dz$



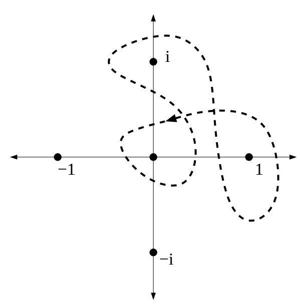
(42) $\int_C \sin(e^z) dz$



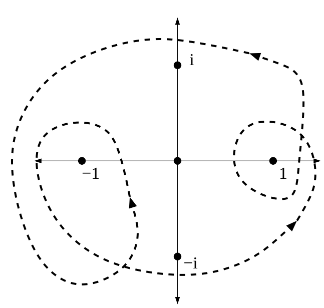
(43) $\int_C e^z dz$



(44) $\int_C \frac{z^2}{z^2+1} dz$



(45) $\int_C \frac{dz}{z^2-1}$



46-50: Answer each question.

(46) Write the function $f(z) = z^3$ in the form $u(x, y) + iv(x, y)$, with u, v real-valued.

(47) Give an example of a function that is analytic on the whole plane except for the points $z = i$ and $z = -i$.

(48) Give an example of an entire function that never equals 1.

(49) Is the function $u(x, y) = x^2 + y^2$ harmonic? Explain why or why not.

(50) Evaluate $\int_C \exp(2z)z^{-4}dz$ where C is the postively oriented unit circle.