MAT 331 Fall 2017, Homework 3
Due in class Thursday, October 19, 2017
Area of the Mandelbrot set

The Mandelbrot set is the set of complex numbers c so that the sequence defined by

$$z_0 = 0, \qquad z_{n+1} = z_n^2 + c$$

has absolute value bounded by 2 for all n. `MATLAB` functions for drawing this set were discussed in class and can be downloaded from the class webpage. The exact area of the Mandelbrot set is unknown, and the purpose of this homework is to estimate this area.

The idea is is that if a set M is inside a rectangle R with dimensions $a \times b$, and we choose a point uniformly at random in the rectangle R, then the chance that it lands in M is area(M)/area(R). Thus if we choose N random points in R and K of them land in M we can deduce area$(M) \approx (K/N)$area(R). This is called the "Monte Carlo" method.

(1) Write a `MATLAB` function to use a for loop to choose N random points in $R = [-2, 2] \times [-2, 2]$ and for each point run check the definition of the Mandelbrot set for S steps (N and S should be inputs of your function). Output the esimated area.

(2) For $N = 1,000,000$ make a plot of the estimated areas for $S = 10, 100, 1000, 10000$.

(3) We could save time if did not need to iterate points that we already know are in the Mandelbrot set, such as points in the main cardioid and the attached disk. The main cardiod are the complex values c so that

$$|1 - \sqrt{1 - 4c}| < 1$$

and the attached disk are the values such that

$$|c + 1| < 1/4.$$

Re-write the code from part (1) so that the iterations are only performed if c is outside both regions. Also make the rectangle smaller, but still containing the Mandelbrot set (use the drawing program to find an appropriate rectangle). Does the new code run faster than (1)? If so, by how much?

(4) If you wanted to test $N = 10^9$ random values with step value $S = 10000$, estimate the time this would take using the methods from (1) and (3) (Run experiments with smaller values of N, say $N = 10^4, 10^5, 10^6, 10^7$ and plot the times needed by each method. Use these to estimate the timed needed for the larger experiment.)