MAT 331 Fall 2017, Homework 1
Computing $e = 2.71828182845904553488480814849 02\ldots$

From calculus, we know that the exponential function $\exp(x) = e^x$ can be computed in several ways, e.g.,

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = \lim_{n\to\infty} \sum_{k=0}^{n} \frac{x^k}{k!}, \tag{1}$$

$$e^x = \lim_{n\to\infty} (1 + \frac{x}{n})^n. \tag{2}$$

Choose some values of x, say $x = 1$, and compute e^x using each of the formulas above for $n = 1, 2, \ldots 10$.

Compare the answers to the value given by the MATLAB function `exp(x)` Make a plot of the differences. Plot the logarithm of the differences. Which approximation goes to zero faster as a function of n? You may need to use the commands `digits` and `vpa` to get enough accuracy.

Compare Equation (2) to

$$e^x = \lim_{n\to\infty} (1 + \frac{x}{2^n})^{2^n}. \tag{3}$$

Explain why these approximations might be especially fast to compute on a binary computer.