MAT 331 Fall 2017, Project 4
Gauss versus Clenshaw-Curtis

This project is based on the 2008 paper "Is Gauss quadrature better than Clenshaw-Curtis?" by Lloyd N. Trefethen.

Gauss quadrature estimates the integral of a function by sampling it at $n+1$ optimally chosen points (the zeros of the Legendre polynomials) and is exactly correct for polynomials of degree $2n$. Clenshaw-Curtis quadrature uses samples at the Chebyshev points and is only exactly correct for polynomials of degree up to n, but Trefethen points out in this article that for most functions it is almost as good as Gauss quadrature and is faster when n is large. This project consists of numerically verifying these claims in some special case, replicating experiments described in Section 3 of the paper.

(1) In Section 2 of his paper, Trefethen gives short (7 line) `MATLAB` functions that implement the two methods. Copy these and verify that they work. Plot the nodes for both methods and $n = 32$ as in Figure 1 of the paper. Explain how you did this.

(2) Recreate Figure 2 in the paper: for the six given functions apply the two methods to compute the integrals and compare them to actual answers. You do not need to recreate the solid lines that show theoretical error bounds. Explain how you fiund the "real" answers in each case. Remember that the plots are for $n = 1, \dots 30$ and the y-coordinate is logarthmic.

(3) For the function $f(x) = 1/(1+16x^2)$, where both methods seem to give similar accuracy in Figure 2 of the paper, make a plot of how long each calculation takes as a function of n. (Use the `MATLAB` commands `tic`, `toc`.)

(4) Recreate Figure 3 of the paper. Also include a table of how long each computation took.

(5) Discuss the claims made in Trefethen's paper and how these are supported by the experiments. Comment on whether your results agree with his or not. If not, can you explain why?