MAT 331 Fall 2017, Project 1
Gauss-Jacobi quadrature and Legendre polynomials

This project is based on my notes "Gauss-Jacobi quadrature and Legendre polynomials" at
`http://www.math.stonybrook.edu/~bishop/classes/math331.F17/Projects/gauss.pdf`

You should read this over, but you need not understand the proofs of all the lemmas and theorems; these are included for completeness, but understanding them is not needed to perform this computational project. Formulas and page numbers below refer to this document.

(1) Approximate the following integrals using Simpson's rule, as done in class:
$$2 = \frac{\pi}{2}\int_{-1}^{1} \cos(\frac{\pi}{2}t)dt, \qquad e - \frac{1}{e} = \int_{-1}^{1} e^t dt.$$
Make a table for the approximate values for $n = 5, \ldots, 20$ and plot the logarithms of the errors (the absolute value of the difference between the approximation and the known, exact value).

(2) Consider the interval $I = [a, b] = [-1, 1]$. Take n equally spaced points in I and use Formula (2) to compute the weights $\{w_k\}$ for these points. Use these weight and points to approximate the same integrals as above (the integrand in (2) is a polynomial so you could use the built-in command `polyint`). Make a table for the approximate values for $n = 5, \ldots, 20$ and plot the logarithms of the errors, and compare these to the results for Simpson's rule.

(3) Use Formula (3) to compute the first 20 Legendre polynomials. For each of these, find all the roots of each polynomials (you can use `fzero` or `roots`; save the roots of the kth polynomial as the kth row of a matrix). For each polynomial and each root use Formula (2) to compute the corresponding weight.

(4) Using the results of the previous step, approximate the same two integrals as before. Plot and log-error for all three types of approximation. How do they compare?

(5) Repeat the experiments for $\int_{-1}^{1} \exp(-x^2)dx$ and make a table of approximations. This integrand doesn't have a a closed form anti-derivative, so we don't have an exact formula for the value of the integral, but `MATLAB` has a built in function `erf` that you can use to calculate the integral. Look in the documentation and use `erf` to compute the integral and compare your approximations to this value

Your project should be a short report that accomplishes each of these goals and should incluide the code you wrote and any tables or grpahs that were requested. It should be written neatly in comprehensible English and full sentences.