

PROBLEM SET 5, MAT 324, Fall 2015
Due Thursday, October 29, 2015 Lebesgue Integration

1. Compute the integral of the Cantor function $\int_0^1 F(x)dm$ (the standard “stair-step” function that is constant on the complementary components of the Cantor middle thirds set).
2. If $f_n(x) = \sin^2(nx)$ find both $\int_0^{2\pi} \liminf_{n \rightarrow \infty} f_n(x)dm$ and $\liminf_{n \rightarrow \infty} \int_0^{2\pi} f_n(x)dm$.
3. What is $\lim_{n \rightarrow \infty} \int_{-\infty}^{\infty} x^n e^{-n|x|}dm$? Find the limit and prove it is correct.
4. Suppose $\{f_n\}$ is a sequence of functions that converges almost everywhere to a function f and define $F_n = \sup_{k=1, \dots, n} |f_k|$. Show that if the integrals of F_n remain bounded as $n \rightarrow \infty$ then $\lim_n \int f_n dm = \int f dm$.
5. Show that $\sum_{n=1}^{\infty} \frac{1}{n} \cos^n(2^n x)$ converges for a.e. x .